Ontario

Ministry of Education

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GRADE 7 GEOMETRY

NOTES FOR TEACHERS

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The resource notes in this module are related to the Grade 7 Geometry Strand for Intermediate Division Mathematics 1977, Draft Copy. They are intended for use by teachers and board curriculum committees as they plan the mathematics program for their schools.

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GRADE 7 GEOMETRY

SECTION 1: BASIC NOTIONS IN GEOMETRY

RELATED SECTIONS AND TOPICS

PAST	FY:	Pages 7, 12
	Ed PJ Div:	Pages 74-76

PRESENT AND FUTURE: This section refers to the vocabulary of geometry and to concepts that are basic to its study. These notions are fundamental to all other geometry sections in Grade 7 and later. These notions could be integrated with topics from the other sections rather than developed as a separate section.

a) Use of geometric terms in the correct context

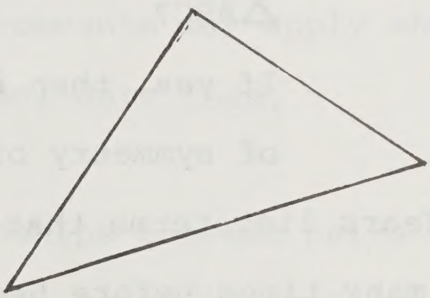
Geometry is an integral part of the mathematics program for Grades K - 6. Teachers of Grade 7 should review both the program content as described in The Formative Years on pages 7 and 12 and the rationale as given in Education in Primary and Junior Divisions on pages 74 to 76. They should also consult with teachers of the K - 6 program about the types of experiences the students have had with geometry — both content and classroom practice (topics, types of investigations, properties investigated, vocabulary, concrete materials used, and sample tests. The local program planned for Grade 7 should grow from the students' earlier experiences, and consolidate and extend the ideas that have been introduced in previous years.

From a formal viewpoint, the vocabulary of geometry is very extensive and precise. It would be a mistake in Grades 7 and 8 to place excessive emphasis on the mastery of exact definitions of terms. Students tend to acquire the meaning of mathematical terms from the things they hear and see every day. Thus it is important for teachers to show by example the use of geometric terms in their correct contexts. In this way, students will refine their understanding of terms already introduced in the K - 6 program. When the occasion warrants it, teachers should assist students directly in this refinement of terms; formal lessons on the exact meaning of many terms at one time should not be necessary.

Students in Grades 7 and 8 are usually familiar with most of the basic geometric terms through a variety of experiences and activities in the K - 6 program. For example, the student usually has used the term point to describe the intersection of two lines, the vertex of an angle, a position on a page or in space, or the sharp 'corner' of a cube, etc. From the context in which it is used, the student learns that a point identifies an exact position (whether on the page, in space, on a map, or in the environment). Whether he/she learns that a point is an undefined term (and the subtleties of distinctions between undefined and defined terms) is not important at this time.

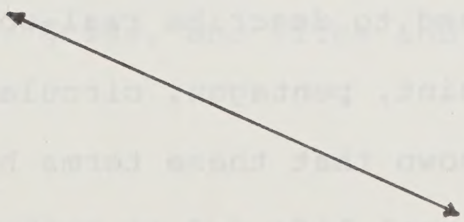
Throughout the study of mathematics, whenever possible, students should first meet and use geometric terms incidentally by such statements as:

i)



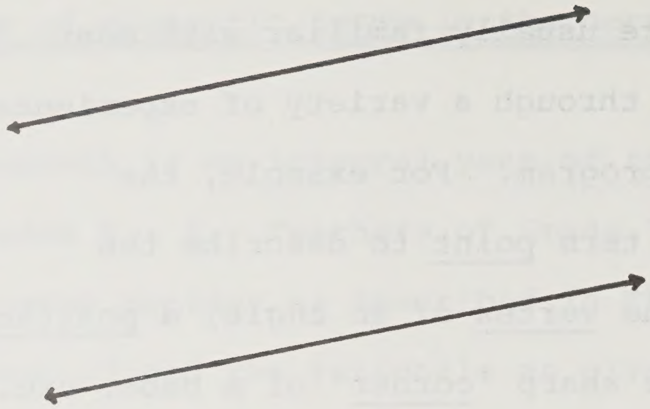
This is a triangle

ii)



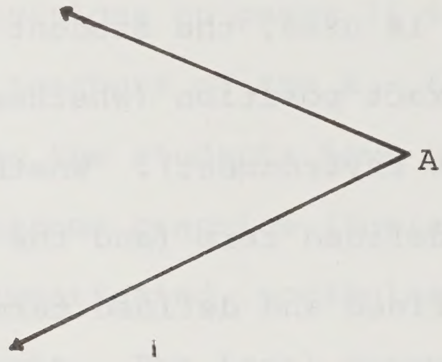
A line extended forever;
it does not have end points

iii)



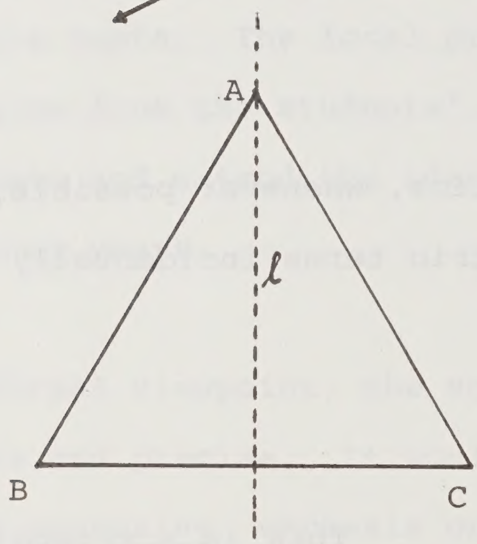
These lines are parallel, they don't meet.

iv)



Point A is the <u>vertex</u> of the angle.

v)



. Make a tracing of $\triangle$ ABC and ℓ.

. Flip the tracing over ℓ.

. Does it still fit onto $\triangle$ABC?

. If yes, then ℓ is a line of symmetry of $\triangle$ABC.

Pages 7 and 12 of The Formative Years list terms that students should have experienced many times before beginning Grade 7.

Many <u>mathematical terms</u> are used to describe real-world objects and situations -- parallel, point, pentagon, circular, for example. By discussion, it should be shown that these terms have <u>similar</u> meanings when used in real life and in mathematics - but that these meanings are not always exactly the same.

b) <u>Intersecting, perpendicular, and parallel lines; properties</u>

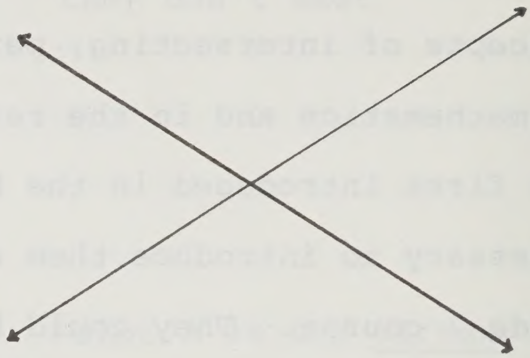
Numerous activities and examples illustrate the meaning of concepts of intersecting, perpendicular, and parallel lines in mathematics and in the real world. Since these concepts are first introduced in the K - 6 program, it may not be necessary to introduce them as an identifiable topic in the Grade 7 course. They could be reinforced, however, as part of many topics that follow this year and in later courses -- symmetry, classification, properties of figures, accurate constructions, transformations, and so on. Students in Grade 7 should learn some techniques for constructing perpendicular lines and parallel lines, as well as some tests for perpendicular and parallel lines. This incorporates part of 7G 2a).

In the event that these concepts are new to the students, the following comments may apply when planning lessons to introduce and/or consolidate them.

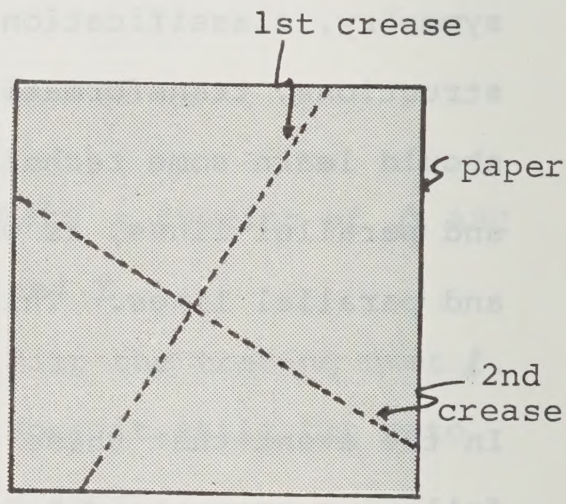
i) Relationships between pairs of lines may be investigated by activities using materials such as: compass, ruler, protractor, parallel rule, tracing paper in conjunction with paper folding, flat mirror, transparent mirror, geoboard, dot paper, grids, and tiles and tiling patterns.

ii) The following illustrate some of the properties that may be discovered through the above materials:

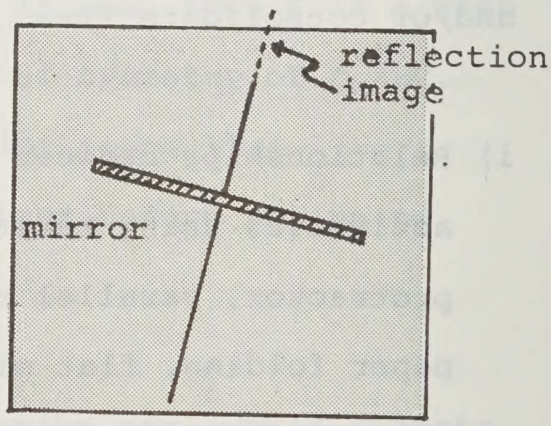
1) Two lines intersect. How are the opposite angles related? The relationship may be found through the use of a protractor, paper folding, tracing paper (flip, half-turn), transparent mirror.



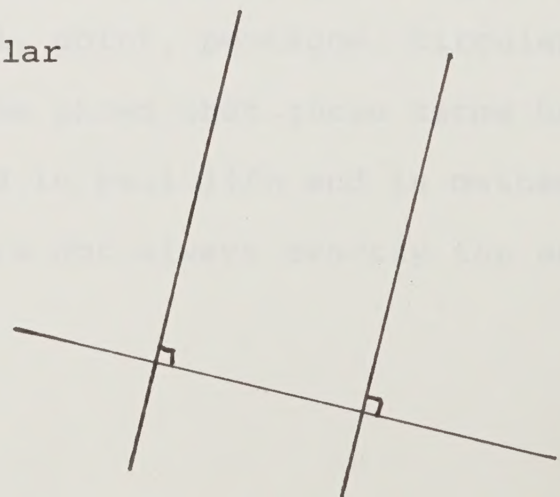
2) A sheet of paper is folded once, then again with the first crease along itself. How are the creases related?



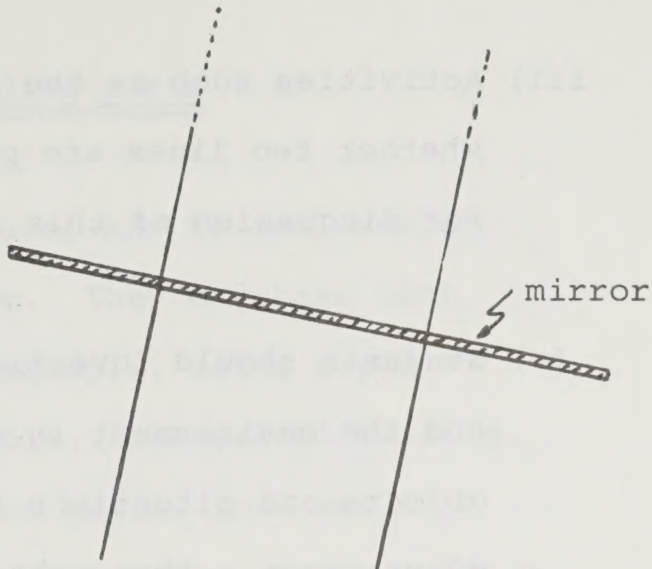
3) A mirror is adjusted so that a line reflects onto itself. How is the mirror related to the line?



4) Two lines are perpendicular to the same line. How are they related?



5) A mirror is adjusted on two lines so that they both reflect onto themselves. How are the lines related?



6) A line is drawn to pass through some of the points on a grid. Find a rule to draw a line through A that is exactly

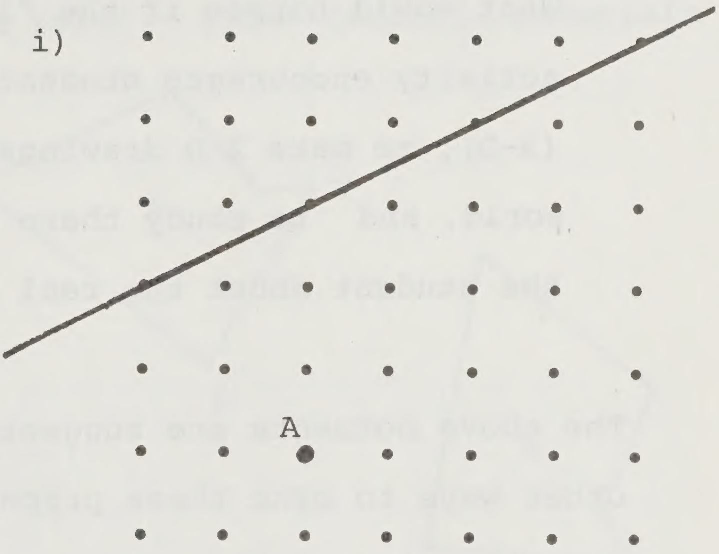
i) parallel

ii) perpendicular

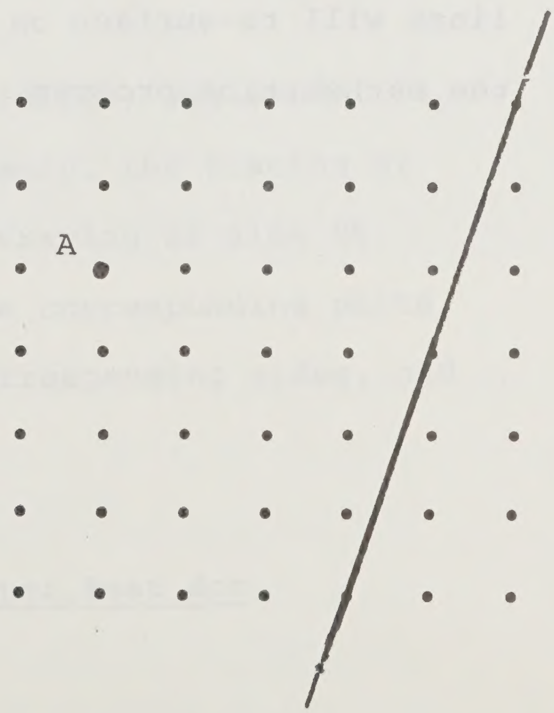
to the first line.

(Or, do this activity on a geoboard.)

i)



ii)



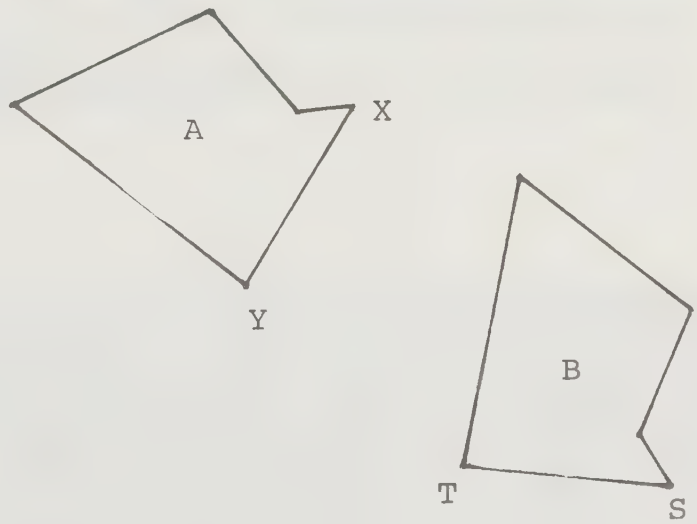
iii) Activities such as the above suggest ways of testing whether two lines are perpendicular, or are parallel. For discussion of this, see the notes for Grade 8 G 1a).

iv) Students should investigate the classroom, rooms at home, and the environment in general, for many examples of objects and situations in which parallel and perpendicular edges occur - then make drawings to represent these. The question should be explored as to why parallel and perpendicular "lines" occur so often in the environment. What would happen if the "lines" were not so? This activity encourages students to look at the real world (3-D), to make 2-D drawings that represent aspects of the real world, and to study these drawings to see what they tell the student about the real world situations.

The above comments are suggestive only; there are numerous other ways to make these properties clear. Any initial investigation of these concepts should not be exhaustive. The concepts of perpendicular, parallel, and intersecting lines will re-surface on numerous occasions throughout the mathematics program.

c) Symmetries of plane figures, corresponding parts

Ideas of symmetry related to plane figures and to real world objects are a part of the K - 6 program. They may have been investigated in a number of ways; for example, using paper folding techniques, carbon paper, mirrors, transparent mirrors, geoboards, ink-blots, dot paper, squared paper, real world examples, or patterns with tiles and with blocks. These earlier experiences should form the base from which a more comprehensive understanding of symmetry is developed. The following notes suggest a development using tracing paper. Other concrete materials may be used for parts of this development.



If a tracing of a figure A fits exactly onto figure B, then figures A and B are congruent -- same shape, same size.

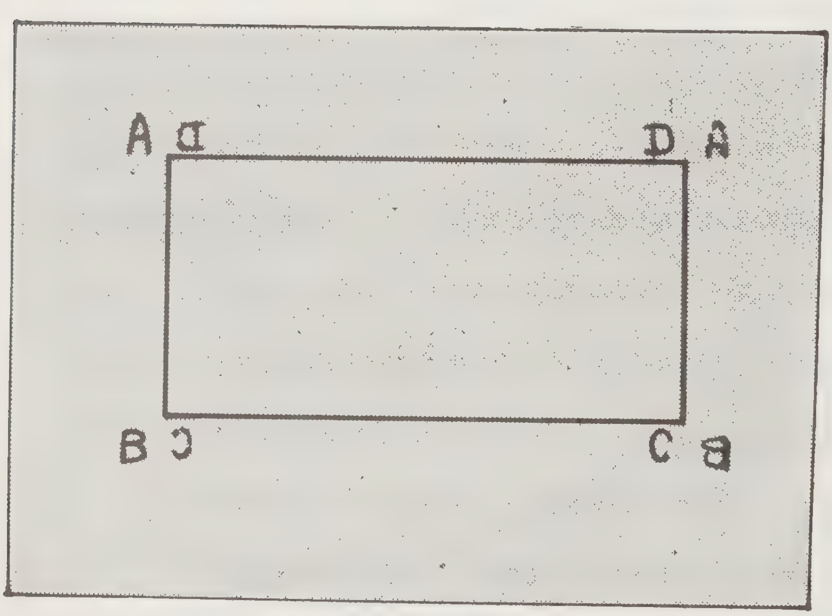
When this happens, corresponding parts (sides, angles) of the two figures are congruent. For example, the tracing of figure A fits exactly on figure B; the tracing of side XY fits exactly on side ST — XY and ST are corresponding parts and are congruent; and so on for all corresponding sides, and all corresponding angles.

The above procedure uses the Tracing-Paper Test for Congruent Figures.

There are two kinds of symmetry that play an important role in the development of geometry: line-symmetry, and rotational symmetry (of which point-symmetry or half-turn-symmetry is a special case).

Line-symmetry probably occurs first in the child's experience, through mirror activities, paper folding, activities with pattern blocks, geoboard activities, or observations of symmetry in the real world.

Tracing paper may be used to show that a figure has one or more lines of symmetry. Trace the figure accurately. Then turn the tracing over and try to fit the tracing exactly onto the figure. Each fitting indicates a line-symmetry.



tracing paper

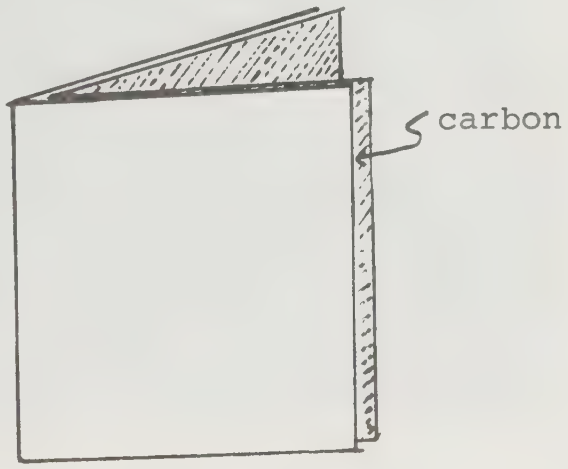
tracing paper, flipped and fitted onto the figure.

The actual line of symmetry can be found by the method below.

Fold so that A fits onto A (B onto B). Crease. Fit the tracing back onto ABCD as in the diagram. Prick two holes (with compass point) in the crease. Draw the line of symmetry through the holes.

There are many activities using a variety of concrete materials that may be used to reinforce the concept of line-symmetry.

. carbon paper is folded carbon-side out and placed in a folded sheet of paper; draw a figure on the outside, observe the images on the inside;



. observe a figure and its image in a mirror, or in a transparent mirror;

. use a transparent mirror i) to construct a line or lines of symmetry; ii) to construct a line-symmetric figure;

. construct line-symmetric figures on dot paper or on a grid;

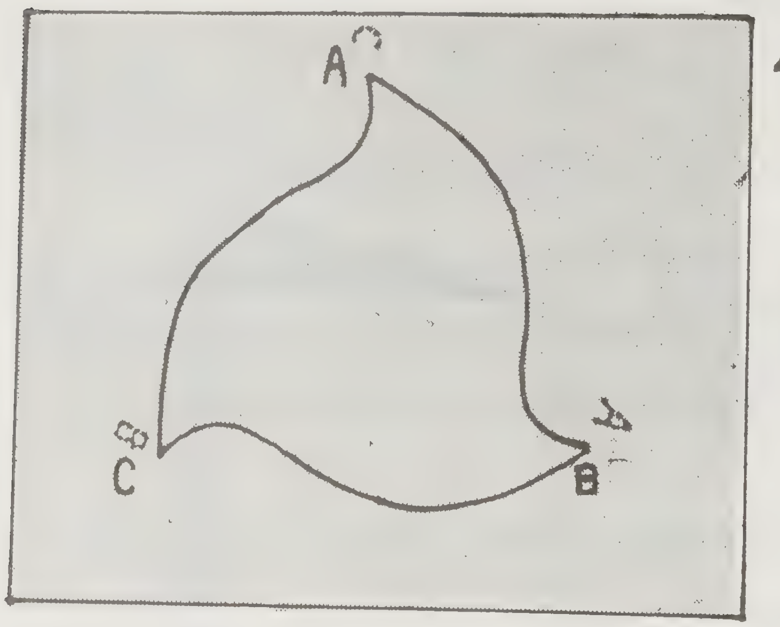
. use prepared grids (or Altair Paper, available from Longman Canada Ltd.) to shade or colour symmetric patterns;

. use paper folding and cut-out patterns;

. use paper folding and pin-prick patterns;

. identify line-symmetry in commercial patterns found in wallpaper, cloth, commercial art, or highway signs, etc.

Rotational symmetry has also been introduced in the K - 6 program.



tracing paper

tracing paper turned through 120^0

To find the centre of the rotation

- fold onto A, crease, prick, draw the line (perpendicular bisector of AB, this is not a line of symmetry of this figure - but the centre of rotation is located somewhere in this line)

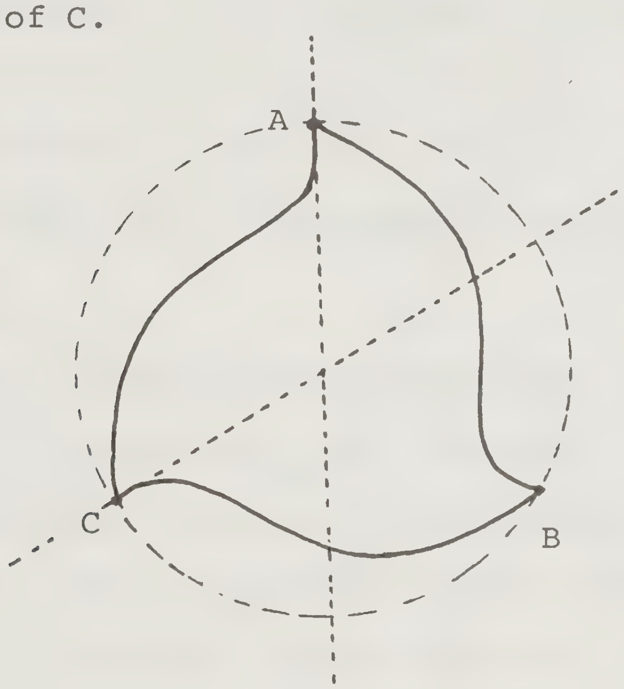
- fold onto C, crease, prick, draw the line - the centre of rotation is located somewhere in this line.
- the intersection of the lines is the centre of rotation.

Place the tracing in its original position; pin the tracing at the centre of rotation; turn the tracing until it fits (in a new position) on figure ABC (as shown).

Will this method work if the figure has only half-turn-symmetry? How would you find the centre of rotation for S?

In the above example, note that the tracing of point A moves in a circular arc about the centre of rotation until it fits onto B (likewise B onto C, and C onto A). This can be demonstrated as follows:

- label the centre of rotation O
- draw a circle with centre O, radius OA
- fit the tracing onto figure ABC
- pin the tracing on O (with compass point, or pencil point)
- turn the tracing about O and watch the path of point A, of B, of C.



Students could be given a variety of figures that have rotational symmetry (drawn on rectangular dot paper, equilateral dot paper), and asked to locate the centre of symmetry. then they could be asked to make a tracing and demonstrate the rotational symmetries of each figure.

Students could be asked to observe figures in the environment that have rotational symmetry, and make sketches (or accurate drawings) of them. A collection could be made on the bulletin board.

Some figures have both line-symmetry and rotational symmetry. This is discussed below. In mathematical terms, a figure F has symmetry if a transformation t exists that maps the figure onto itself; i.e. $t(F) = F$. This formal definition of symmetry is beyond the expectations for the Intermediate Division. However, the 'tracing-paper test for symmetry', as described below, is within the grasp of students at this stage and is a concrete demonstration of the above concept.

The paperback Reflections and Rotations contains many excellent activities related to symmetry, that are ready for use in the classroom at this stage. It is available from Oakland Schools, 2100 Pontiac Lake Road, Pontiac, Michigan 48054, in student and teacher editions. Ask about other materials.

The tracing-paper test can be used

i) to discover whether a figure has symmetry;

ii) to determine the number and kinds of symmetries;

iii) to identify the corresponding (congruent) parts of the figure.

For example, with each of the following figures:

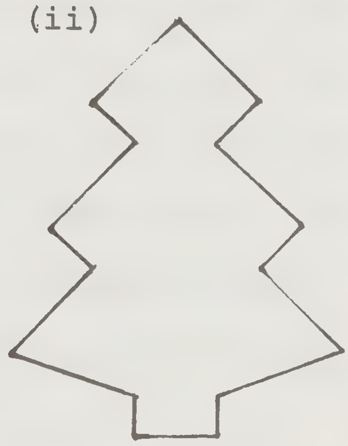
- make a tracing

- fit the tracing onto the figure as often as possible. (if necessary flip the tracing paper)

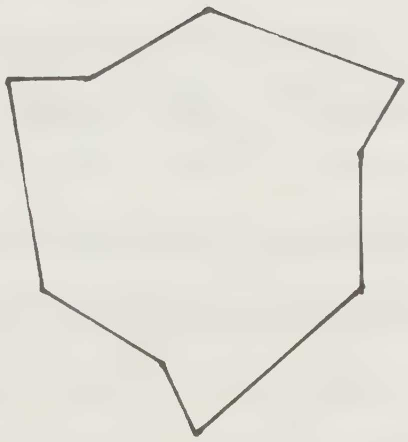
- each fitting illustrates a symmetry of the figure, and the corresponding (congruent) parts under the symmetry.

(i)

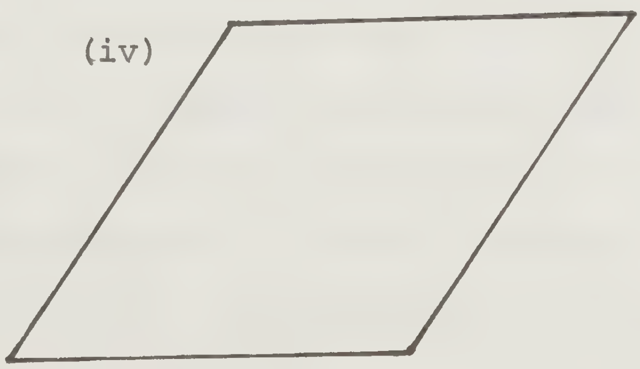
(ii)



(iii)



(iv)



In the above example, the tracing fits figure

i) in one new way (a turn-symmetry);

ii) in one new way (a flip-symmetry);

iii) in two new ways (two turn-symmetries);

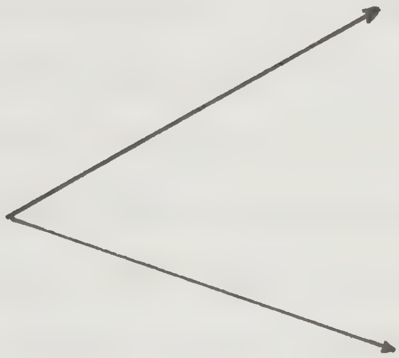
iv) in three new ways (one turn, two flips)

Teachers should be aware that each tracing also fits the figure in its original position. This is the trivial case (a turn-symmetry of 360^0 about any point, the identity mapping). This observation is not significant for students at this time; however, some students may ask whether or not this indicates a symmetry of the figure. The answer is yes, in the strict sense. If a student brings up the point, it should be recognized as a very keen observation. In later courses, where the order of symmetry of a figure is considered, this trivial case is included (figures i) to iv) have order of symmetry 2, 2, 3 and 4 respectively, every plane figure has rotational symmetry of order 1 -- a rotation of 360^o about any point).

Again, a simple recall (or introduction) and reinforcement of the notions of symmetry should be sufficient at this time. Symmetry plays an important role in the development of many geometric ideas that follow in this and other courses. There will be many opportunities to use these concepts in developing other topics in the future. There are many other activities that may be used to investigate and consolidate ideas of symmetry (using materials such as geoboards or transparent mirrors).

d) Classification of angles, triangles, and quadrilaterals

Angles



An angle is a figure formed by two rays with a common end point (vertex). Students should be familiar with the terms ray, arm, vertex, size, and measure. This topic should be developed as part of 7G 2c).

The size of an angle is the number of degrees that one ray turns to fit onto the other. The turn may be demonstrated with tracing paper. The size of the angle can be obtained with a protractor.

Angles are classified by their size: acute ($< 90^0$), right (90^0), obtuse ($> 90^0, < 180^0$), straight (180^0), reflex ($> 180^0$). This vocabulary can be introduced incidentally over a period of time by activities related to angles, triangles, quadrilaterals, and polygons in general.

Triangles

A triangle is a closed figure formed by three segments.

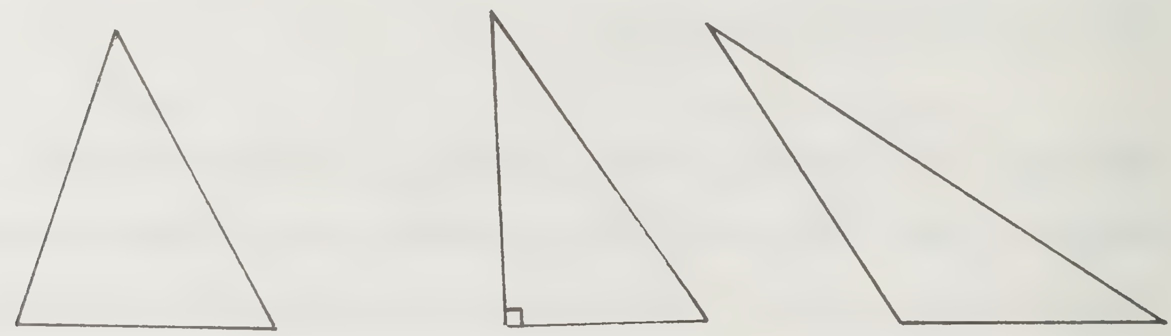
Triangles may be classified by

i) the size of the largest angle;

ii) the lengths of the sides;

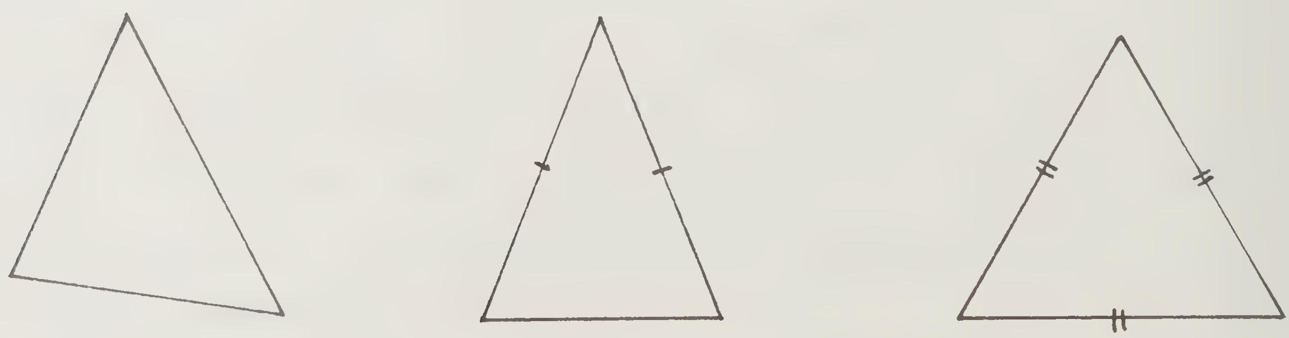
iii) the number of lines of symmetry.

In i), triangles are called acute angled, right angled, or obtuse angled.



acute-angled triangle right-angled triangle obtuse-angled triangle

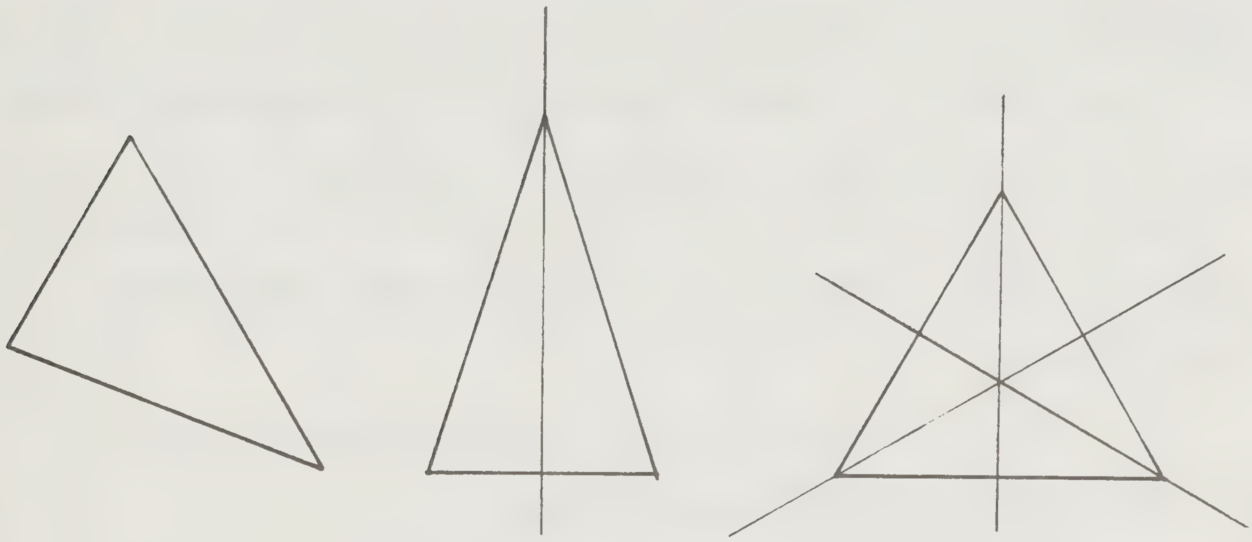
In ii), triangles are called scalene, isosceles, or equilateral if they have 0, 2, or 3 equal sides respectively.



scalene triangle isosceles triangle equilateral triangle

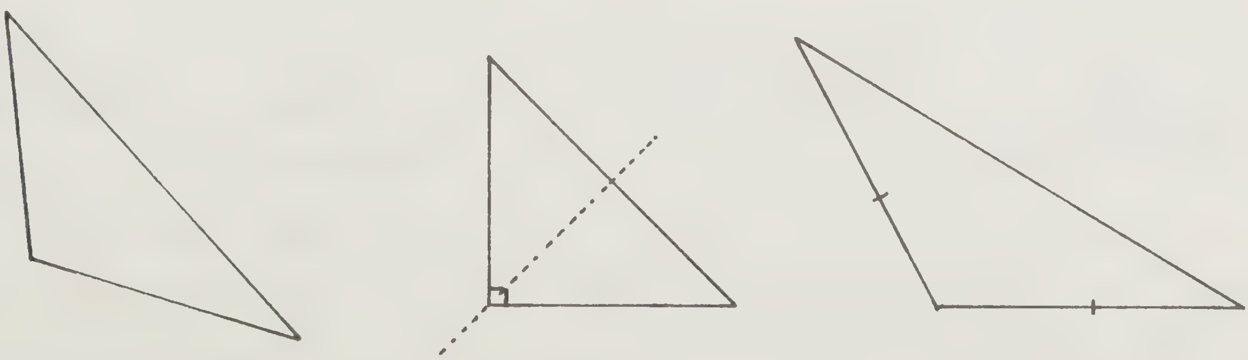
This classification scheme is discussed in most books, and can be investigated by measurement, and by the use of tracing paper, transparent mirrors, or paper folding.

In iii), triangles are called scalene, isosceles, or equilateral depending on the number of lines of symmetry. This classification can be investigated by using tracing paper and folding the tracing, mirrors (visual suggestion) or transparent mirrors.



scalene triangle   isosceles triangle   equilateral triangle

Students should know all three ways of classifying triangles. Exercises with dot paper (square, equilateral) or grids provide good reinforcement. Triangles are frequently named by a combination of the above classifications; for example,



obtuse-angled scalene triangle   right-angled isosceles triangle   obtuse-angled isosceles triangle

Classifying by line-symmetry is very useful for determining the properties of the different types of triangles. See the notes for 8G 3a)

Quadrilaterals

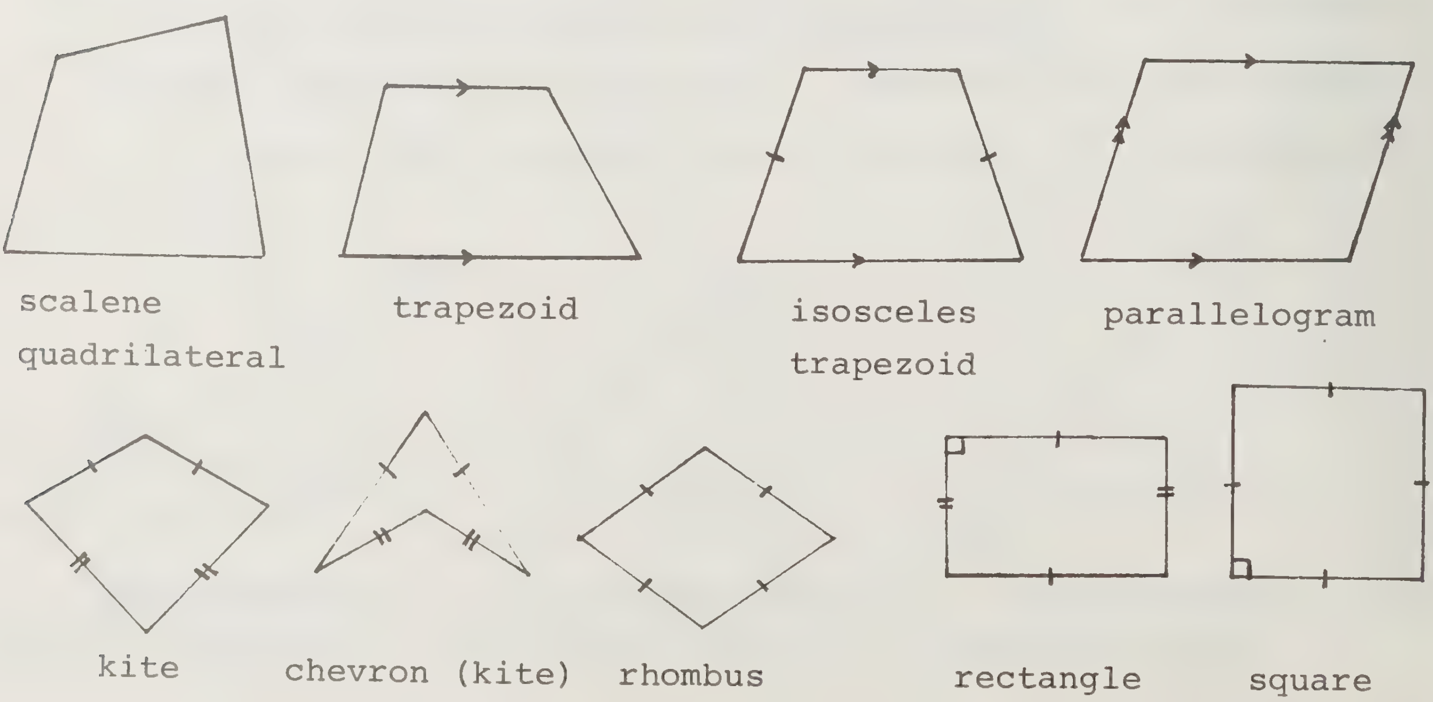
Any attempt to classify quadrilaterals formally becomes quite a complex procedure, in which the various classes of the quadrilaterals are considered as subsets and/or intersection sets of other classes. This is not the intention of this section. At the Grade 7 level, we are looking for a simple set of characteristics by which each type of quadrilateral can be identified.

A quadrilateral is a closed figure formed by four segments. There are nine types, as illustrated below; they may be classified by

i) relations between their sides and angles, or

ii) symmetry and parallel lines

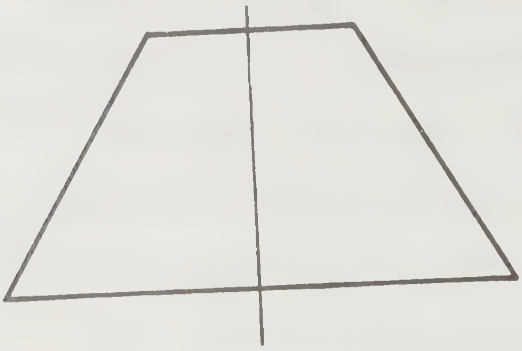
Classification of quadrilaterals by sides and angles is discussed in most books. The diagrams below identify the side-angle properties that identify each type of quadrilateral.



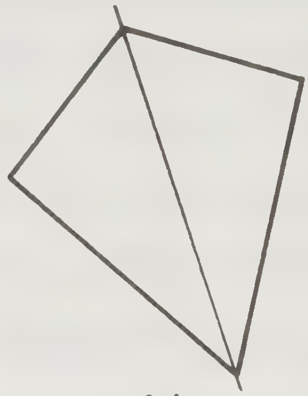
Students could be given a set of quadrilaterals and asked to find side-angle properties by means of ruler and protractor, tracing paper, compasses, transparent mirrors, or other materials. They will likely find many properties beyond those shown in the above diagrams. For example, in the rectangle they may find that opposite sides are parallel and all angles are equal to 90^0. This is good; it shows that a rectangle is a special kind of parallelogram (so is a rhombus). Further, they may observe that i) a square is a rhombus and therefore a parallelogram; ii) a square is a rectangle and therefore a parallelogram. The side-angle properties in the above diagrams are the ones most often used to identify the different types of quadrilaterals.

Classification of quadrilaterals by symmetry-parallelism

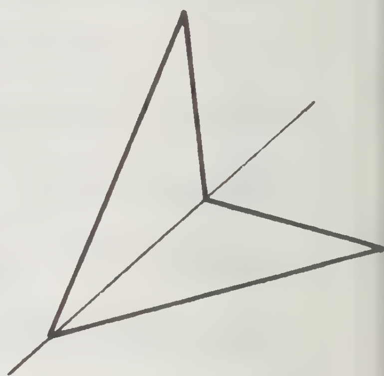
Students could be asked to draw the lines of symmetry of various quadrilaterals; they could use tracing paper (see the notes for 7G 1c)) or a transparent mirror. Some types do not have any. The results are shown below.



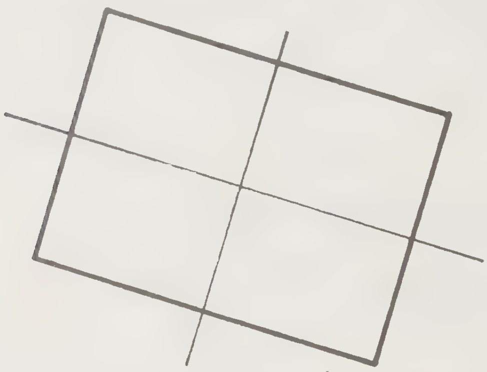
isosceles trapezoid



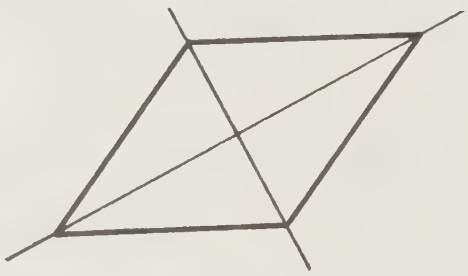
kite



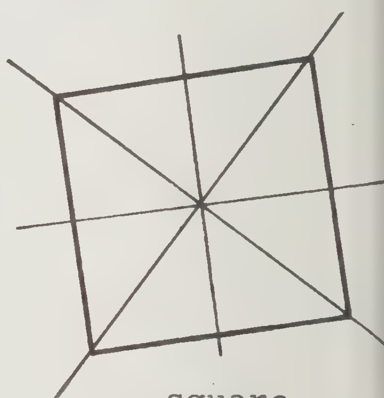
chevron (kite)



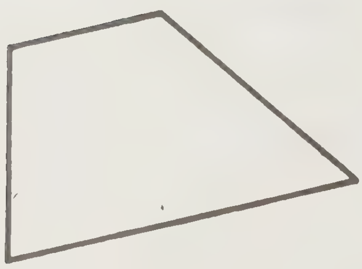
rectangle



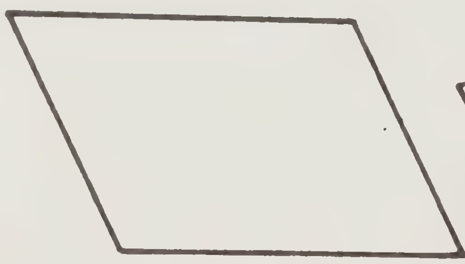
rhombus



square



trapezoid

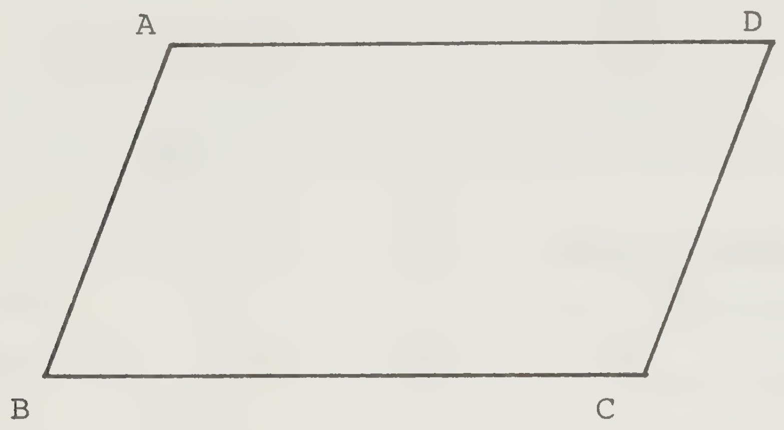


parallelogram

scalene quadrilateral

The lines of symmetry as shown, determine whether a quadrilateral is an isosceles trapezoid, a kite, a chevron (a concave kite), a rhombus, a rectangle, or a square - diagonal lines of symmetry must be distinguished from mid-point lines of symmetry.

The parallelogram does not have line-symmetry. Students may be asked whether it has any other kind of symmetry, then to demonstrate their answer with tracing paper.

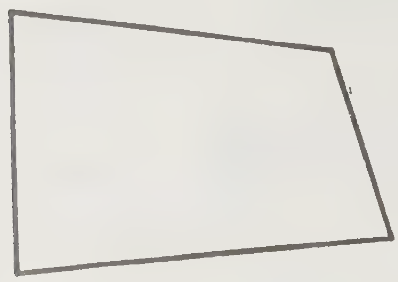


To do so they might turn the tracing half way around, and fit it onto figure ABCD (with A,B,C, and D on C,D,A, and B respectively).

Now students could be asked to locate the centre of the turn i) by guess and test (virtually impossible) and ii) by geometric construction (students should intuitively recognize that the centre is the intersection of the diagonals) and test.

Students could be asked "What other quadrilaterals have half-turn symmetry? Demonstrate this with tracing paper".
This exercise will show that the rhombus, rectangle, and square are special cases of the parallelogram. The half-turn centre is the intersection of the diagonals.

The different quadrilaterals can be identified by the symmetry-parallel properties below.



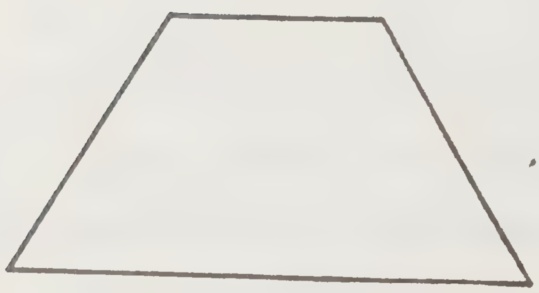
no symmetry

no parallel lines

Scalene Quadrilateral

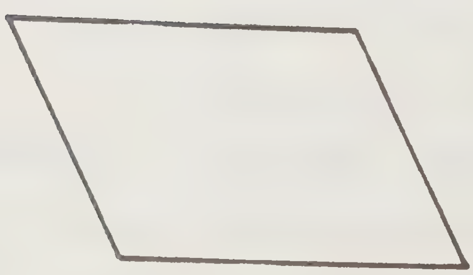
1 pair of parallel lines

Trapezoid



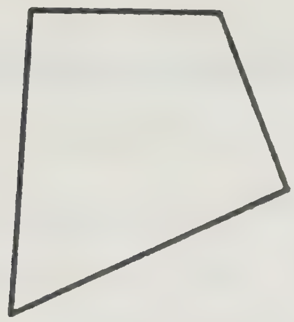
1 mid-points line of symmetry

Isosceles Trapezoid



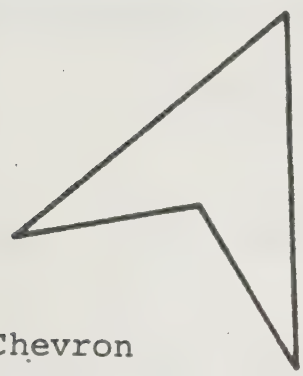
half-turn symmetry

Parallelogram



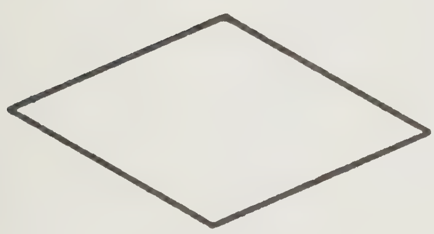
Kite

1 diagonal line of symmetry



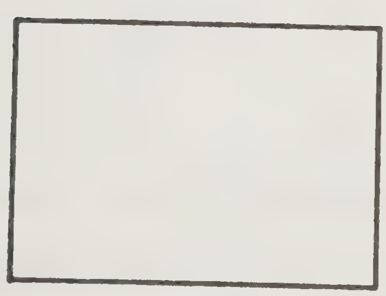
Chevron

1 diagonal line of symmetry
concave (a special case of a kite)



Rhombus

2 diagonal lines of symmetry



Rectangle

2 mid-points lines of symmetry

Square

4 lines of symmetry
(2 diagonal, 2 mid-points)

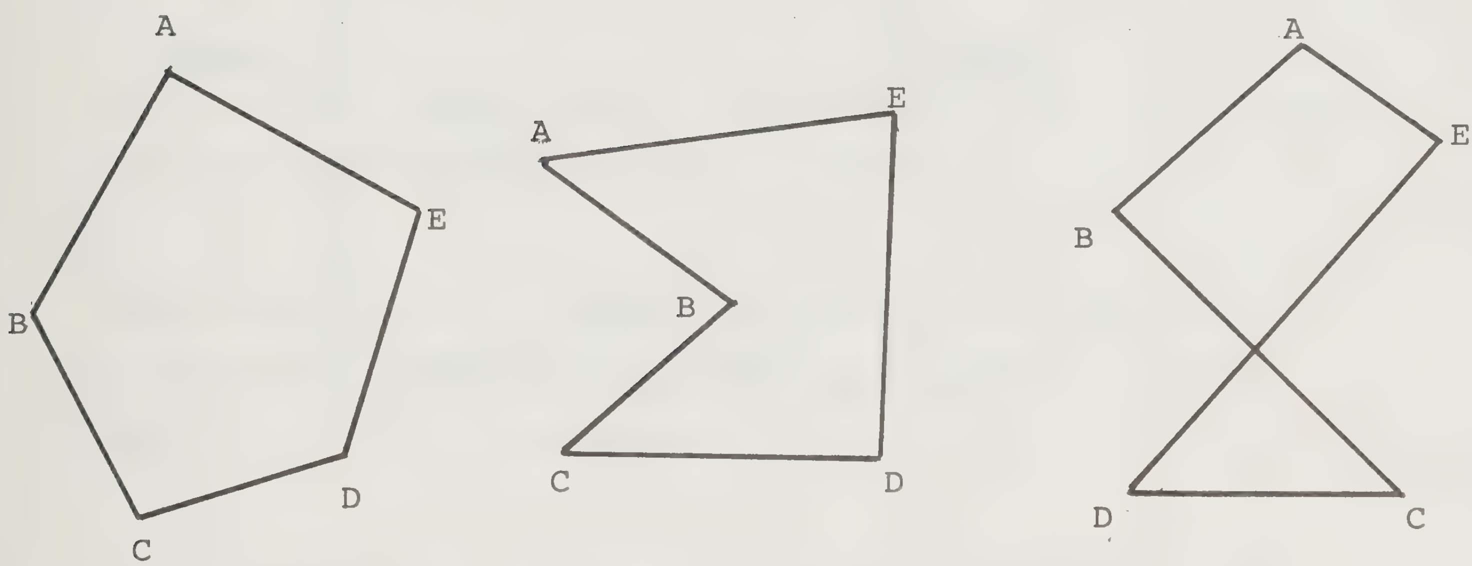
Either classification scheme of triangles and quadrilaterals serves the purpose of identifying and naming the figures by a simple (non-ambiguous) set of properties. It is suggested that students investigate both schemes.

In topic 8G 3a), a complete set of properties for each figure will be developed. At this time, the symmetry properties imply all the congruence properties of the figure.

Students should be asked to use dot paper and/or grid paper to draw and label the different types of triangles and quadrilaterals. Consider the case of the equilateral triangle informally; a special grid is needed.

e) Polygons; symmetry properties of regular polygons

Although this topic is optional, the meaning of polygon and regular polygon should be established at some stage in the program for Grades 7-10. This statement applies only to simple polygons, not to nonsimple polygons, as illustrated below; and principally to convex polygons.

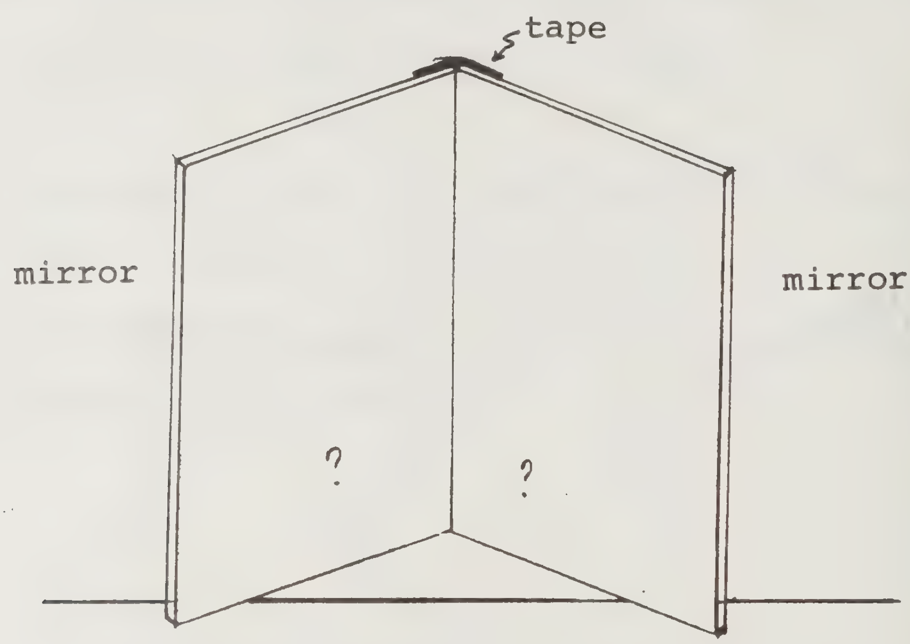


simple polygon (convex) | simple polygon (concave) | nonsimple polygon

Triangles and quadrilaterals are polygons with three and four sides. Pentagons (five sides), hexagons (six sides) and octagons (eight sides) are the most common of the other polygons; they occur frequently in our environment (in nature, in man made designs). Students could be encouraged to make collections of pictures or diagrams of these occurrences. Many commercial designs and logos are essentially polygons.

In a <u>regular polygon</u> all sides are equal, and all angles are equal. The equilateral triangle and square are regular polygons of three and four sides.

A pair of hinged mirrors may be used to investigate the regular polygons and their lines of symmetry.



Stand the hinged mirrors on a line segment, as shown, so that the angle is bigger than 90^0. Adjust the angle until a triangle is observed. Note the apparent shape of the triangle, the size of the angles, the 'lines of symmetry'. Why are all sides equal? All angles equal?

Close the angle between the mirrors until a quadrilateral appears, a pentagon, a hexagon, and so on. Compare the length of the sides, the size of the angles. Observe the lines of symmetry. What types of figures are formed? Explain why.

Make diagrams to represent the above cases. What happens as the angle between the mirrors is made smaller?

Find a relation between the number of sides and the number of lines of symmetry.

Does a regular polygon have rotational symmetry? If yes, how many rotational-symmetries does it have?

Examine the lines of symmetry for regular polygons having 3, 5, 7, ... sides, for regular polygons with 4, 6, 8, ... sides. How are they different?

Note that the lines of symmetry all meet in a point. This is the <u>centre of symmetry</u> (of rotational-symmetry) of the regular polygon. Do all regular polygons have <u>point-symmetry</u>? If not, which ones do and which do not?

Ministry of Education Ontario

GRADE 7 GEOMETRY

SECTION 2: ACCURATE CONSTRUCTIONS

RELATED SECTIONS AND TOPICS

PAST	FY:	Pages 7 and 12
	Ed PJ Div:	Pages 74-76
PRESENT	Gr 7:	N 3c; N 8; G 1cd; G 3e; G 4c; G 5c
FUTURE	Gr 8:	N 3ad; N 6; G 1; G 2ac; G 3abd; G 4b; G 5b
	Gr 9 Gen:	N 2c; N 3c; G 1; G 3
	Gr 9 Adv:	N 2e; G 1; G 3abcd; G 4a
	Gr 10 Gen:	N 4c; N 7; G 1
	Gr 10 Adv:	G 3

a) Constructing congruent segments, congruent angles, perpendicular lines, perpendicular bisector, angle bisector, and parallel lines

In the program for the Intermediate Division it is recommended that the properties of geometric figures should be developed inductively, with each student investigating relationships between the sides and angles, then discussing their findings in groups or in the class. This method requires that the figures be drawn accurately. If the students are to draw their own figures, they will need, therefore, to learn techniques for making accurate construction

The topic of Accurate Constructions is optional in Grade 7 because of limitations of time; it becomes core in Grade 8. There are many techniques for constructing figures accurately, other than the traditional methods using ruler and compass. Some of these are appropriate at this stage, particularly if introduced incidentally as a part of other topics. Some intuitive techniques are suggested below. In Grade 8, students could then review these and extend them to include ruler and compass techniques.

Perpendicular and Parallel Lines

Topic 7G 1b) deals with perpendicular and parallel lines. The student should learn a variety of techniques to test whether two lines are perpendicular or parallel.

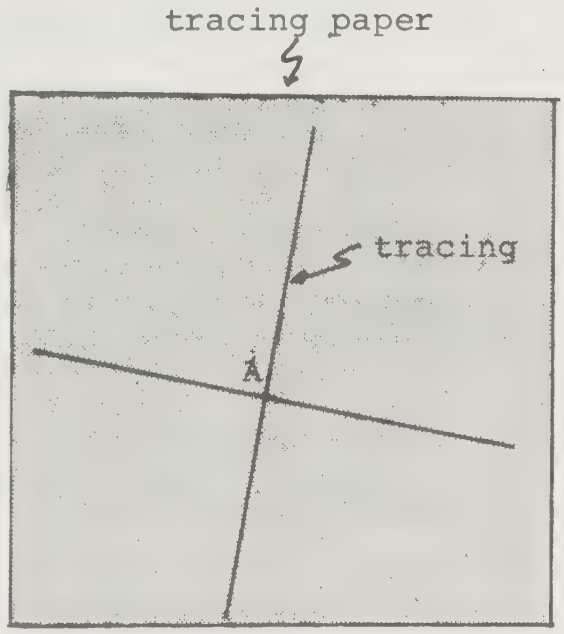
To show that lines are perpendicular, one or more of the following methods may be used:

i) Measure the angle with a protractor (part of topic 7G 2c)).

ii) Try to fit the corner of a rectangular page or card (or a set square) onto the angle.

iii) Use tracing paper, as indicated below.

Trace the lines on tracing paper, then try to fit the tracing in a new position. (If it fits again, the lines are perpendicular; if it doesn't, they are not perpendicular.)



iv) Use a transparent mirror as suggested in the notes for 7G 1b), page 6.

There are other techniques. The students should try to find some for themselves.

Now students can be encouraged to develop techniques for constructing perpendicular lines using variations on the above procedures - protractor, ruler (marked identically on both edges), set square, rectangular card, tracing paper, or transparent mirror. These techniques are useful when dealing with topics 7G 1b)c)d)e), 2b) and 4c). As an example, the use of tracing paper to construct a perpendicular through a point is given below.

Using Tracing Paper to Construct a Perpendicular Through a Point

- Given a segment AB and a point P (in AB, or elsewhere). See figure 1.

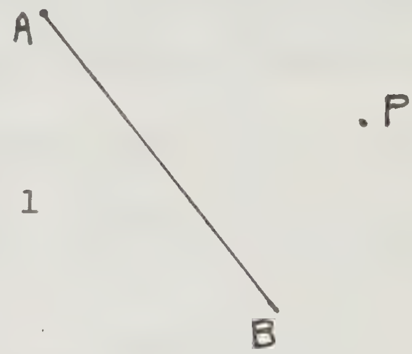


figure 1

- On tracing paper, draw a line m.

- Fold m onto itself. Crease. See figure 2.

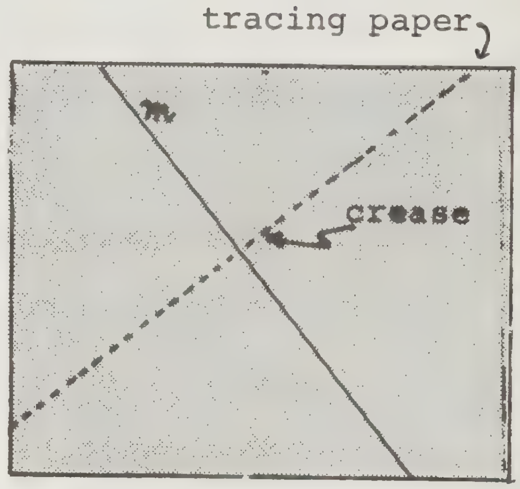


figure 2

- Unfold and turn the tracing paper over

- Fit the crease on AB so that m passes over P. See figure 3.

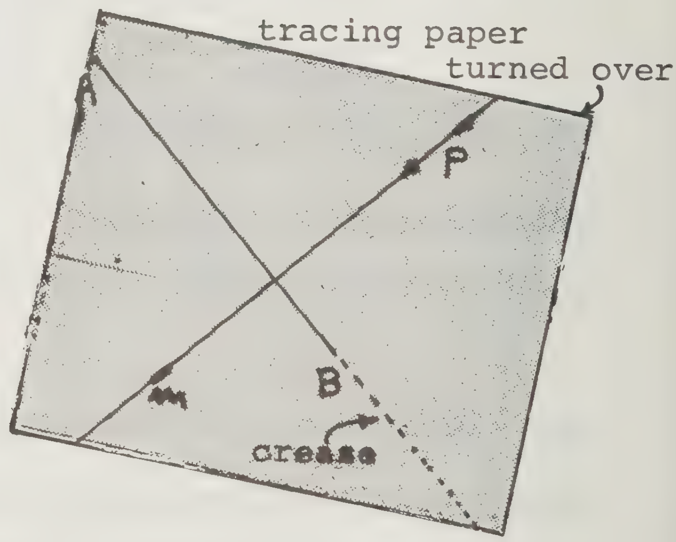


figure 3

- Press hard on any two parts of m. See figure 3.

- Take away the tracing. See figure 4.

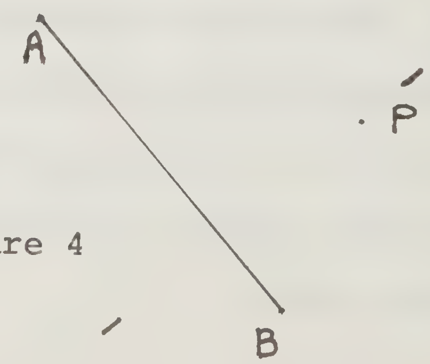


figure 4

- Draw a line through these marks (the required perpendicular).

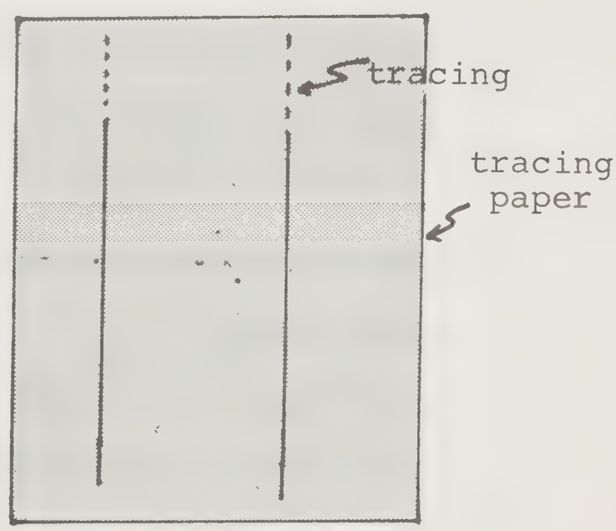
The above technique is simple to use, although lengthy to describe. There are similar techniques for the other constructions.

The following situations might be investigated:

- the importance of perpendicular lines and perpendicular planes in the real world;
- situations in which a person needs to construct perpendicular lines and parallel lines in the home; how are these made?
- the importance of a plumb line. Is it perpendicular to the floor?
- drawings of real objects in which the parts are perpendicular.

To <u>show that lines are parallel</u>, one or more of the following methods may be used:

- Use a ruler or tracing paper to measure the distance between the lines in several positions.
- Trace the lines on tracing paper. Turn over the paper and test whether they still fit.



- Use a transparent mirror, as suggested in the notes for 7G 1b), page 7.
- Construct a perpendicular to one of the lines. Test whether it is perpendicular to the other line.

Now students can be encouraged to develop techniques for <u>constructing parallel lines</u>, using variations on the above procedures; they could, for example, trace both sides of a ruler, copy from lined

paper or grid paper, construct perpendiculars to the same line, use tracing paper, or use a transparent mirror.

As an example, the <u>use of tracing paper to construct a line through a point parallel to a given line</u> is given below.

- Given a segment AB and a point P. See figure 1.

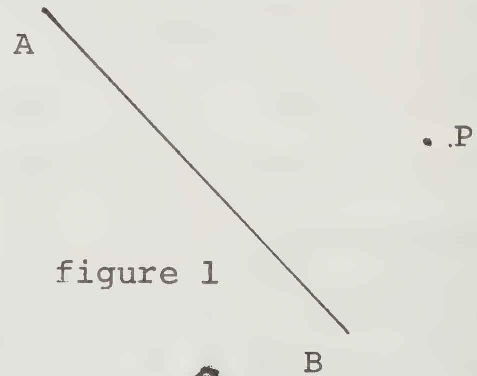


figure 1

- Use tracing paper. Trace AB and P.

- Turn the tracing paper over.

- Fit the tracing of AB along AB, so that the tracing of P is in a new position. See figure 2.

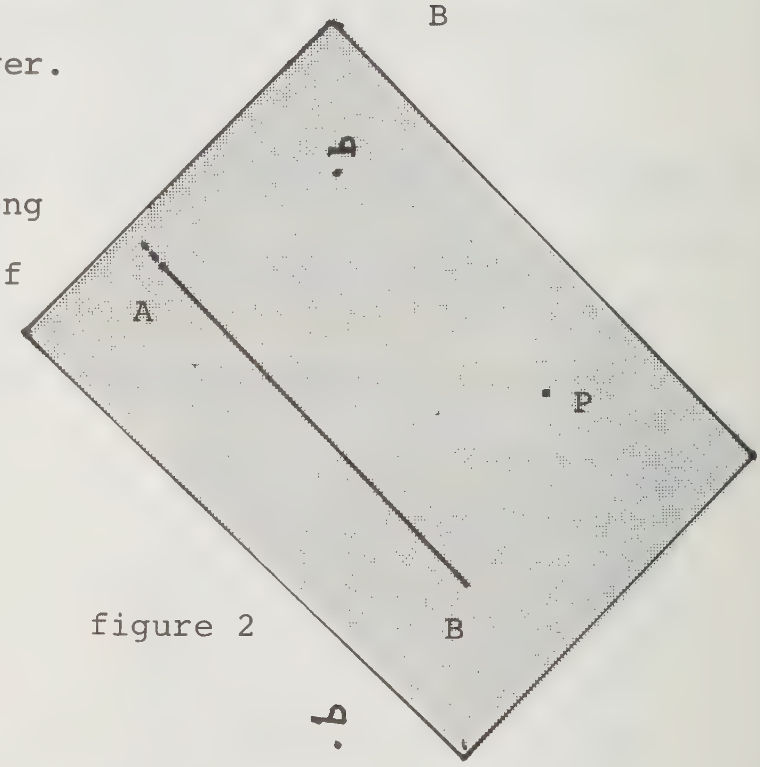


figure 2

- Mark the position of P

- Take away the tracing. See figure 3

- Draw the line through P P (the required parallel line).

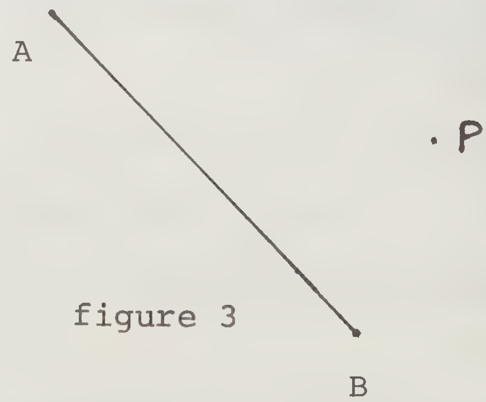


figure 3

The following aspects of this topic could be investigated:

- the importance of parallel lines and parallel planes in the real world;

- the techniques a carpenter uses to construct perpendicular lines and parallel lines (the draftsman);
- the use of a carpenter's level, the meanings of horizontal and vertical;
- parallel rule, its use by the navigator;
- parallel and perpendicular lines in home decorating;
- drawings of real objects in which parallel lines occur.

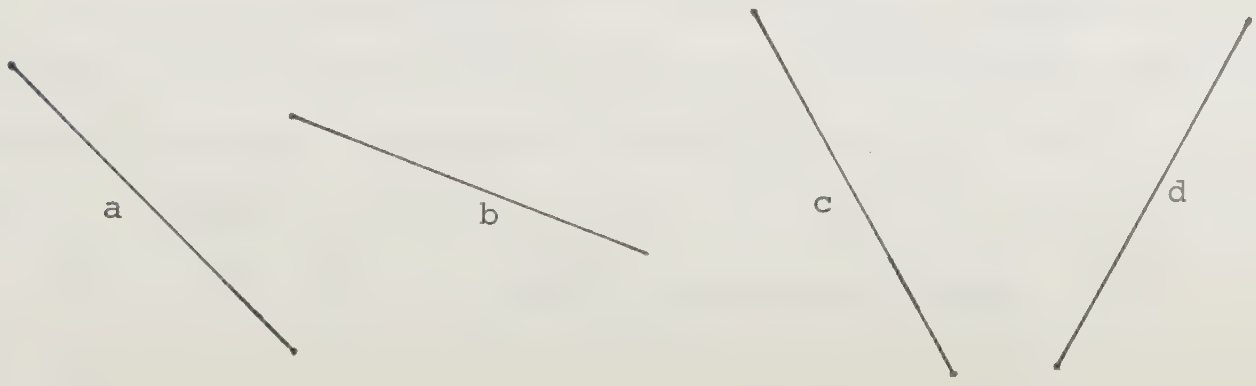
Congruent Segments, Congruent Angles

When investigating properties of figures, it is necessary for the student to have techniques for comparing the lengths of segments and the sizes of angles and/or for measuring these lengths and angle sizes.

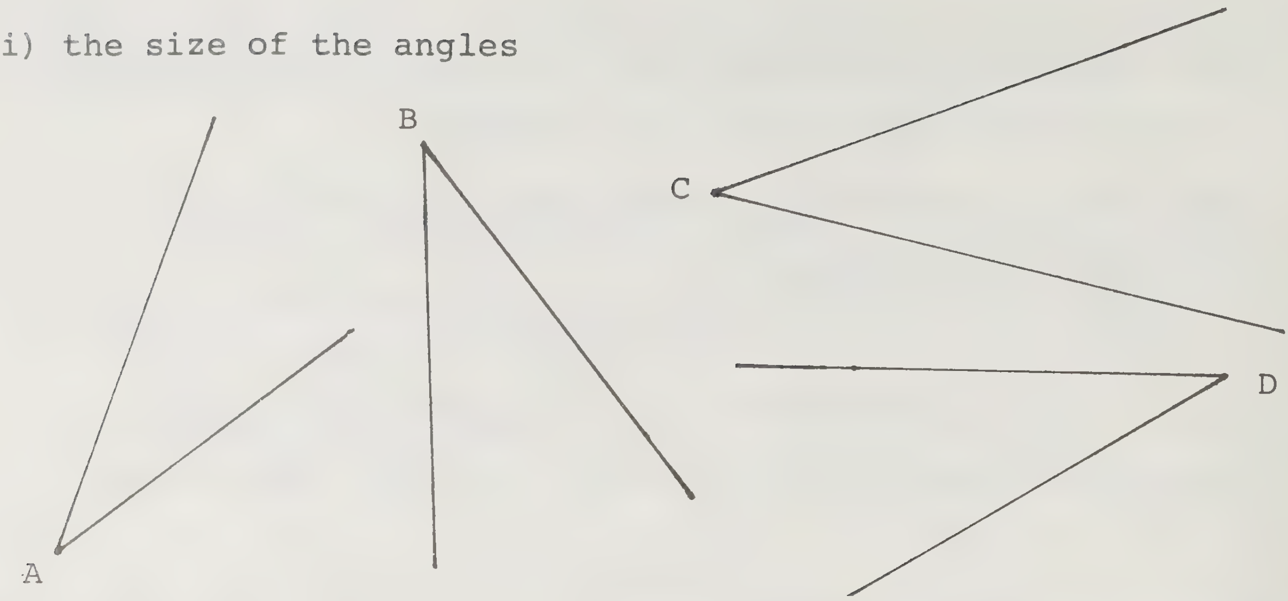
The measurement of segments and angles makes use of rulers and protractors. Techniques for teaching the use of these instruments are commonly known and are not discussed here.

Lengths of segments and sizes of angles can be compared (equal, greater than, less than) by fitting a tracing or mirror image of one onto the other. For example, use tracing paper to compare

i) the lengths of the segments below;



ii) the size of the angles



This technique is significant to the long-range development of geometry through the use of transformations. A tracing of a figure (or a part of a figure) will be moved (flipped over a line, turned about a point, or slid) either

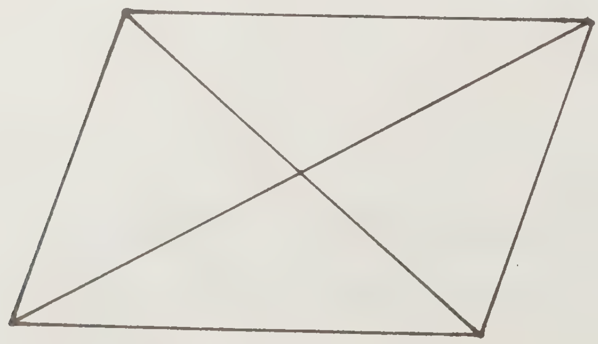
i) to create an image figure;

ii) to fit physically (or not to fit) on a different figure; or

iii) to fit physically onto itself (a symmetry).

These are important stages in developing the strategies for proof based on the use of transformations. For example, make a tracing of the quadrilateral below (parallelogram).

- Show that opposite sides are parallel.
- Trace the figure; pin it at the intersection of the diagonals; half turn the tracing and observe the segments and angles that fit; list the congruent segments, the congruent angles.



Perpendicular Bisector, Angle Bisector

As the names imply, perpendicular bisectors and angle bisectors are lines of symmetry and can be constructed using paper folding techniques, tracing paper, ruler and protractor, transparent mirror, or ruler and compass (if a more formal treatment is desired). See the notes for 8G 1a).

b) Constructing rectilinear figures

This topic involves the accurate construction of triangles, quadrilaterals, and possibly other polygons given certain of their characteristics. It can be extended to include scale drawings of real-world objects (floor plans or furniture designs, for example) and to determine properties of these figures (example, the altitudes of a triangle are concurrent).

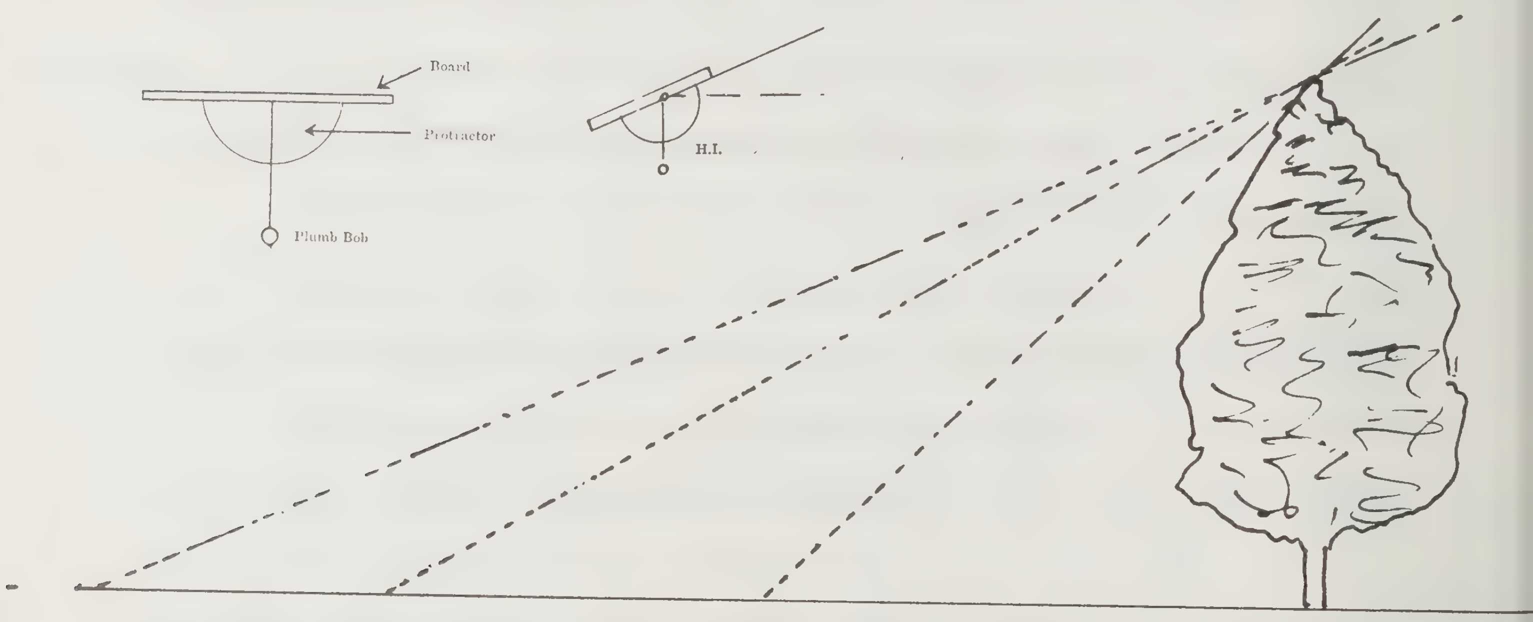
This is an optional topic that provides opportunities to practise the skills and concepts of section 1 and 2a). These ideas are developed more extensively in 8G 2b).

c) Measuring and constructing angles with a protractor; angle properties of various figures

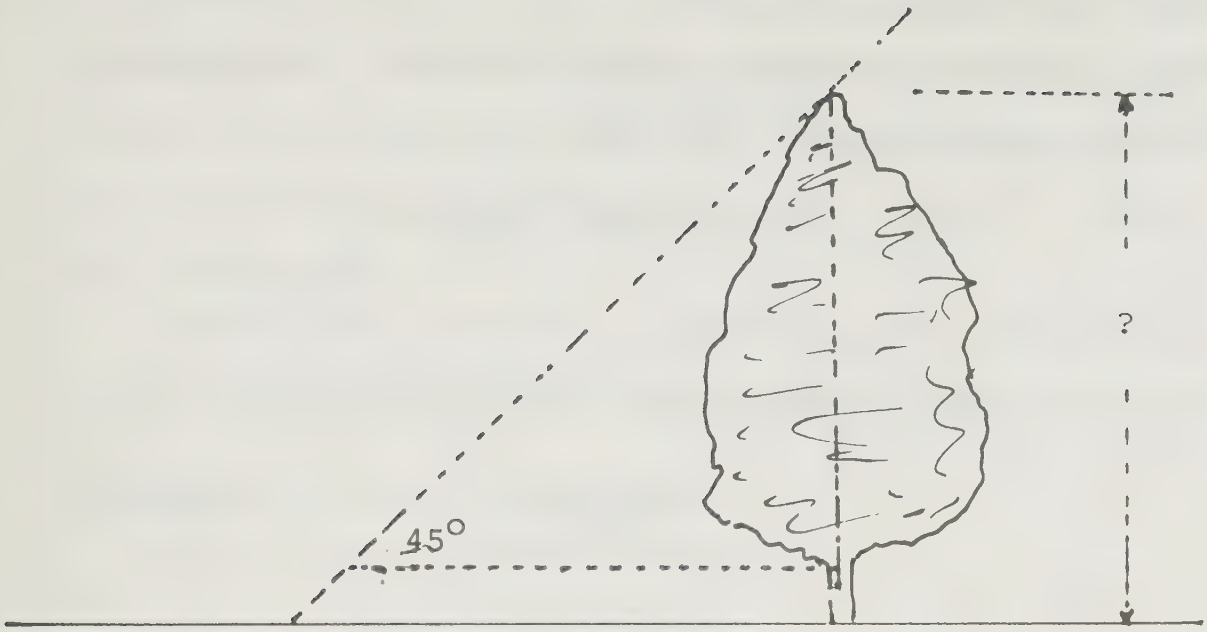
Most students should be familiar with the use of a protractor from their experiences in the K - 6 program. If available, a variety of instruments should be used - semi-circular protractor, circular protractor, blackboard model, clinometer, and a magnetic compass.

There are many opportunities in this and later courses for students to practise their skills with the protractor. Frequent experiences are preferable to a single sequence of lessons.

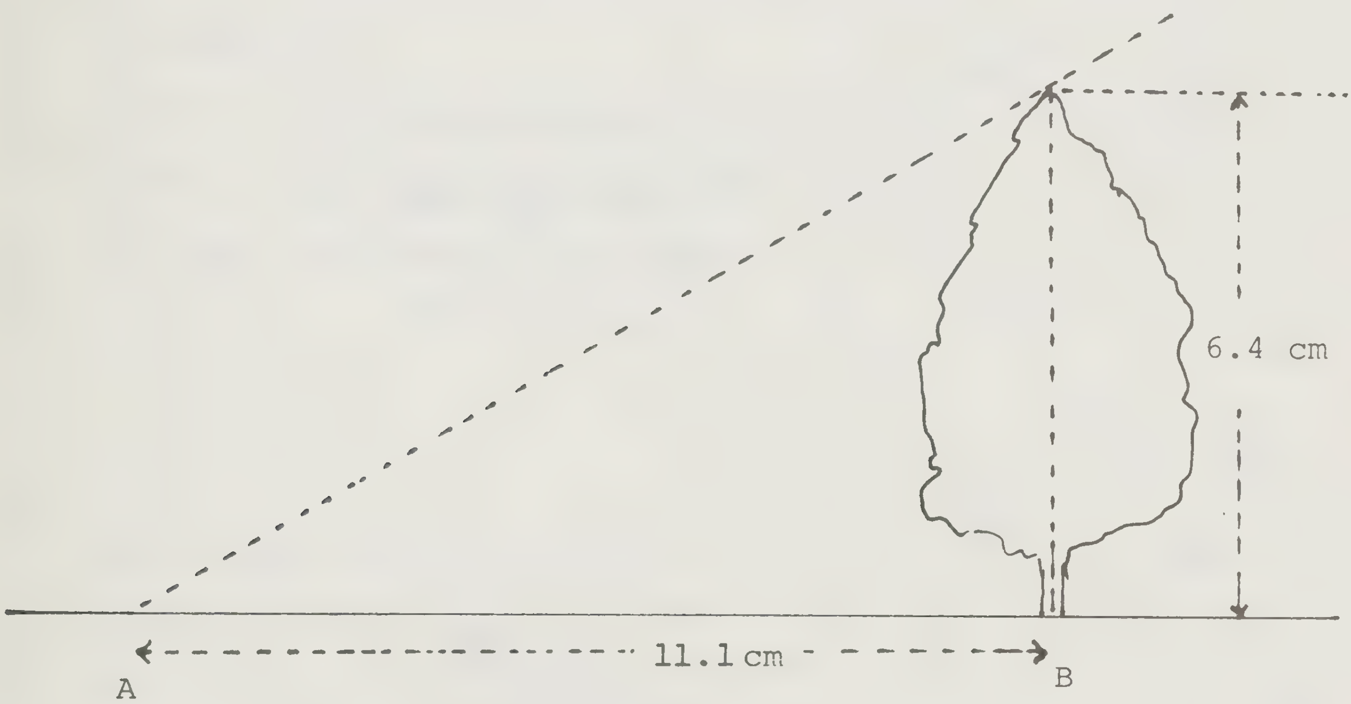
Activities outside the classroom related to large scale measurement should be considered. For example, students could use a classroom protractor and plumb line (or clinometer -- see commercial catalogues) to sight the top of a tree or building at various positions and to measure the angle of elevation at these positions.



Find a position at which the angle is 45°, 60°. etc. How can the position for 45° be used to find the approximate height of the tree?



This topic can be related to the work on scale drawings in 7G 5b), where the scale drawing represents the real world situation.



First, find the <u>scale factor</u>, <u>then</u> the <u>height of the tree</u>.

i) Real world distance from A to B $= 32.5$ m

Diagram distance from A to B $= 11.1$ cm

$= 0.111$ m

<u>Scale factor</u> from diagram to real world $= \frac{32.5}{11.1}$

(pencil & paper arithmetic, approximately) $\doteq 293$

ii) height of tree in diagram $= 6.4$ cm

$= 0.064$ m

<u>height of tree in real world</u> $= 293 \times 0.64$ m

(pencil & paper arithmetic, approximately) $= 19$ m

(Using a calculator, $32.5 \div 0.111 \times 0.064$

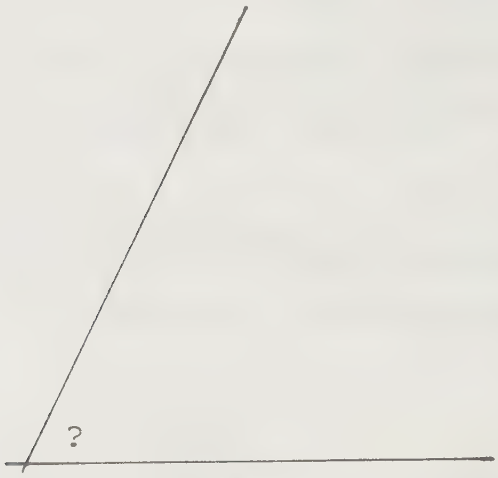
$= 18.738738 \doteq 19$)

It would be worthwhile for students to estimate the size of angles, then check their measurements with a protractor.

	Estimate	Actual

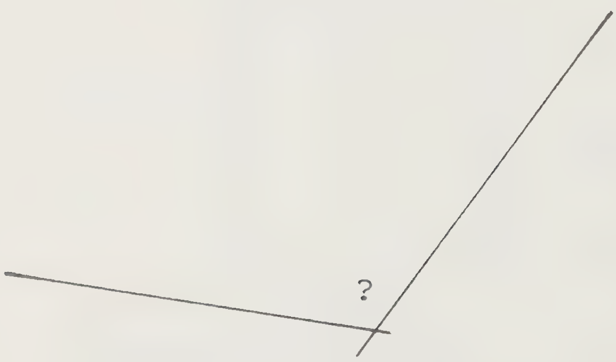
$<90^\circ, >45^\circ$

estimate 70°



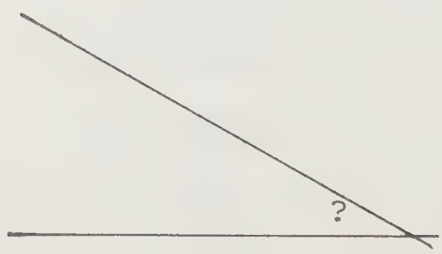
$>90^\circ$ by about 15°

estimate 105°



$<45^\circ$, about $1/3$ of 90°

estimate 30°

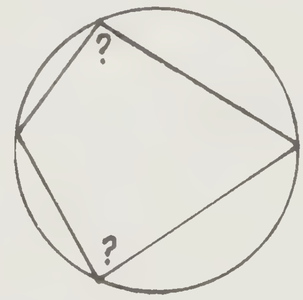


Generally speaking, people have difficulty in estimating the size of an angle. The above exercise will help to refine this skill.

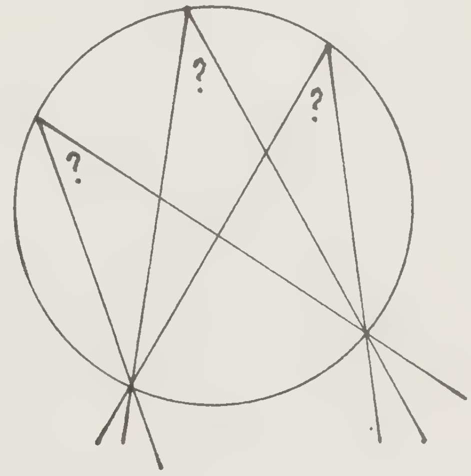
Students could be asked, using a ruler only, to draw an angle of specified size -- say 90°, 60°, or 30° -- then to measure the size of the angle and calculate the error in degrees, and as a percentage.

Students should investigate angle properties of various figures. These could include the following:

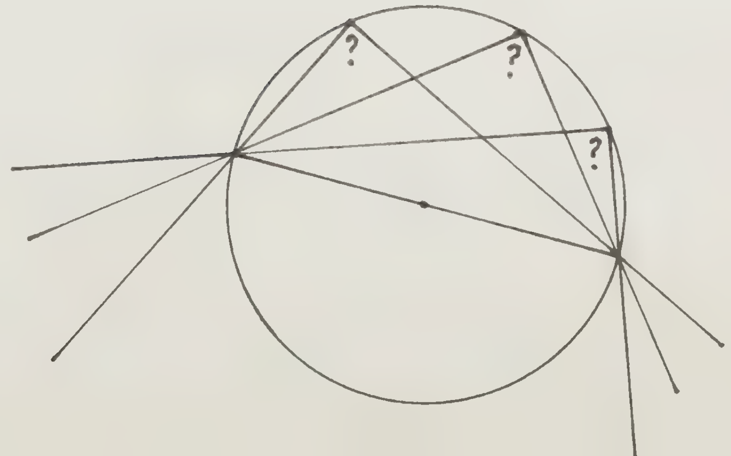
- sum of the angles of a triangle;
- exterior angle of a triangle;
- opposite angles formed by intersecting lines;
- sum of the angles of a quadrilateral;
- sum of the exterior angles of a triangle, a quadrilateral, a pentagon;
- sum of the opposite angles of a cyclic quadrilateral (a quadrilateral inscribed in a circle);



- angles in the same segment of a circle;



- angles in a semi-circle.



Ministry of Education Ontario

GRADE 7 GEOMETRY

SECTION 3: PATTERNS WITH TILES

RELATED SECTIONS AND TOPICS

PAST	FY:	Pages 7, 12
	Ed PJ Div:	Pages 74-76
PRESENT	Gr 7:	N 4acd; N 8cd; A 1b; A 3; G 1; G 2a; G 4; G 5; G 6d
FUTURE	Gr 8:	N 1d; N 6a; A 1b; G 1a; G 2; G 3; G 4
	Gr 9 Gen:	G 3; G 4abd
	Gr 9 Adv:	N 5b; G 1a; G 2a; G 3; G 4
	Gr 10 Gen:	G 1
	Gr 10 Adv:	G 3; G 4

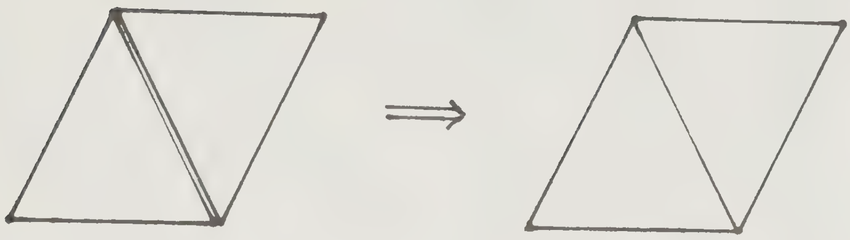
General Comments

Activities with tiles provide a methodology for investigating many properties of geometry. Even though they are optional, it is suggested that they be considered as part of the strategy for developing some of the topics in the Grade 7 Geometry program and in the courses of later grades. The activities are often interesting enough to students to provide high motivational value. Many properties of geometry may be discovered, introduced, or reinforced in an informal setting through these activities.

In the sample activities that are described in the following notes, each student (or group of students) should work with his or her own sets of congruent tiles. These can be prepared easily from master grids that have been printed on cover stock; students can assist by cutting out the tiles. Sample grids are given on the following three pages.

Students should make diagrams to represent the results of the investigations. Such diagrams may be made either by tracing the figures that are formed by the tiles or by using dot paper or grids. Many of the diagrams given in the following notes are intended to illustrate figures that can be formed by two or more tiles; double lines have been used to show common edge(s) between the tiles. The diagrams that the students make should illustrate the mathematical figures that are suggested by the tiles and, when pertinent, the properties of these figures.

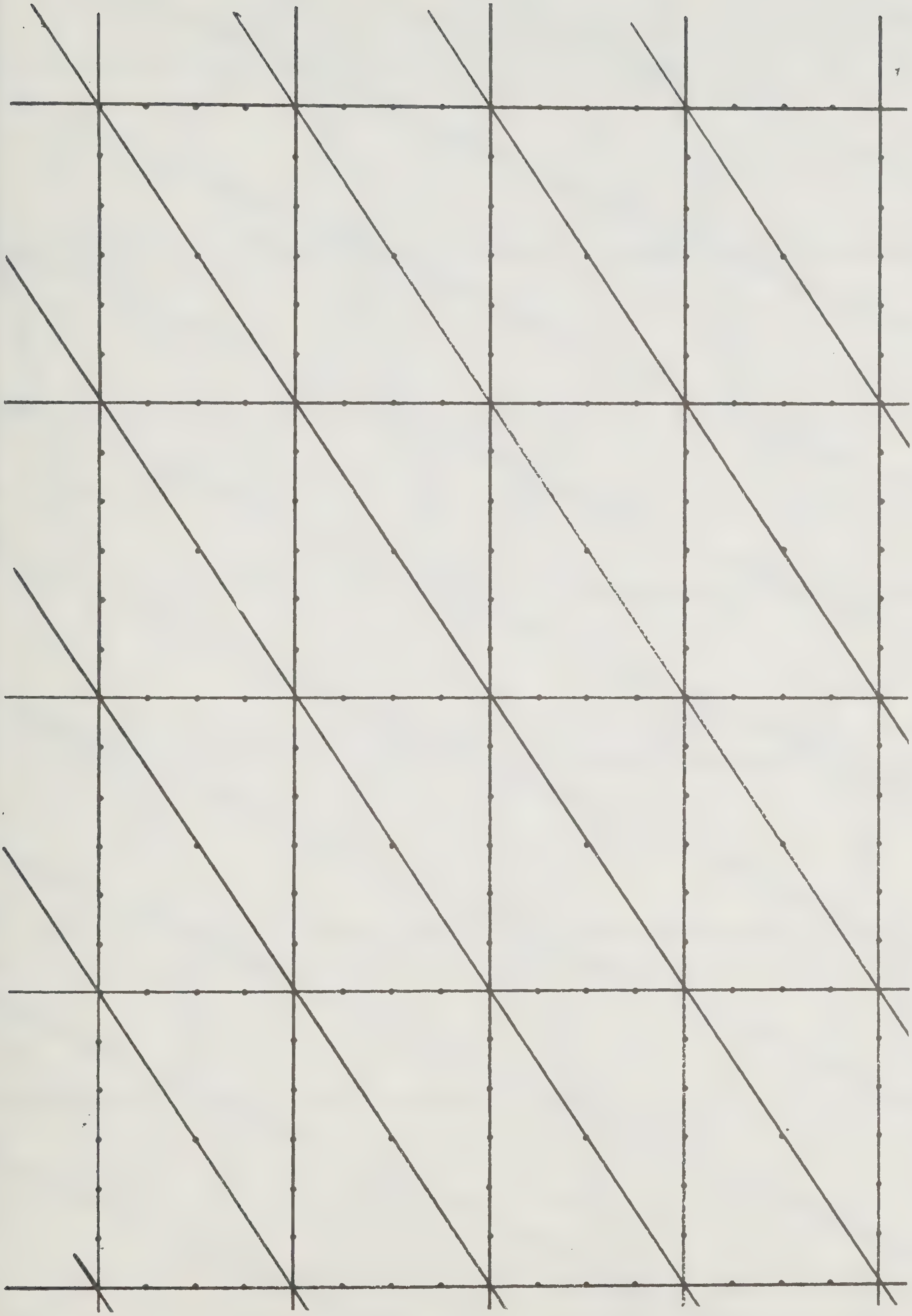
Real World | Mathematical Representation



Pattern of Tiles on Student's Desk | Diagram made in Student's Notes

The same tiles as used by the students can be used on an overhead projector both to introduce the nature of the activities and to discuss the results of the students' investigations. When the images are projected onto the blackboard, their outlines can be traced and their properties labelled directly on the blackboard during class discussion. This process illustrates a real world situation (the pattern of the tiles) being converted into a mathematical representation (the geometric figure).

The notes on tiling patterns that follow apply to the complete geometry program for the Intermediate Division Grades 7 to 10, and are not repeated in the notes for other sections or courses. It should be emphasized that not all of these notes are intended for the present Grade 7 course; they are included here to show the significance of tiling activities to properties that appear both now and later in the program.



a) <u>Fitting two or more congruent tiles; properties of the figures formed</u>

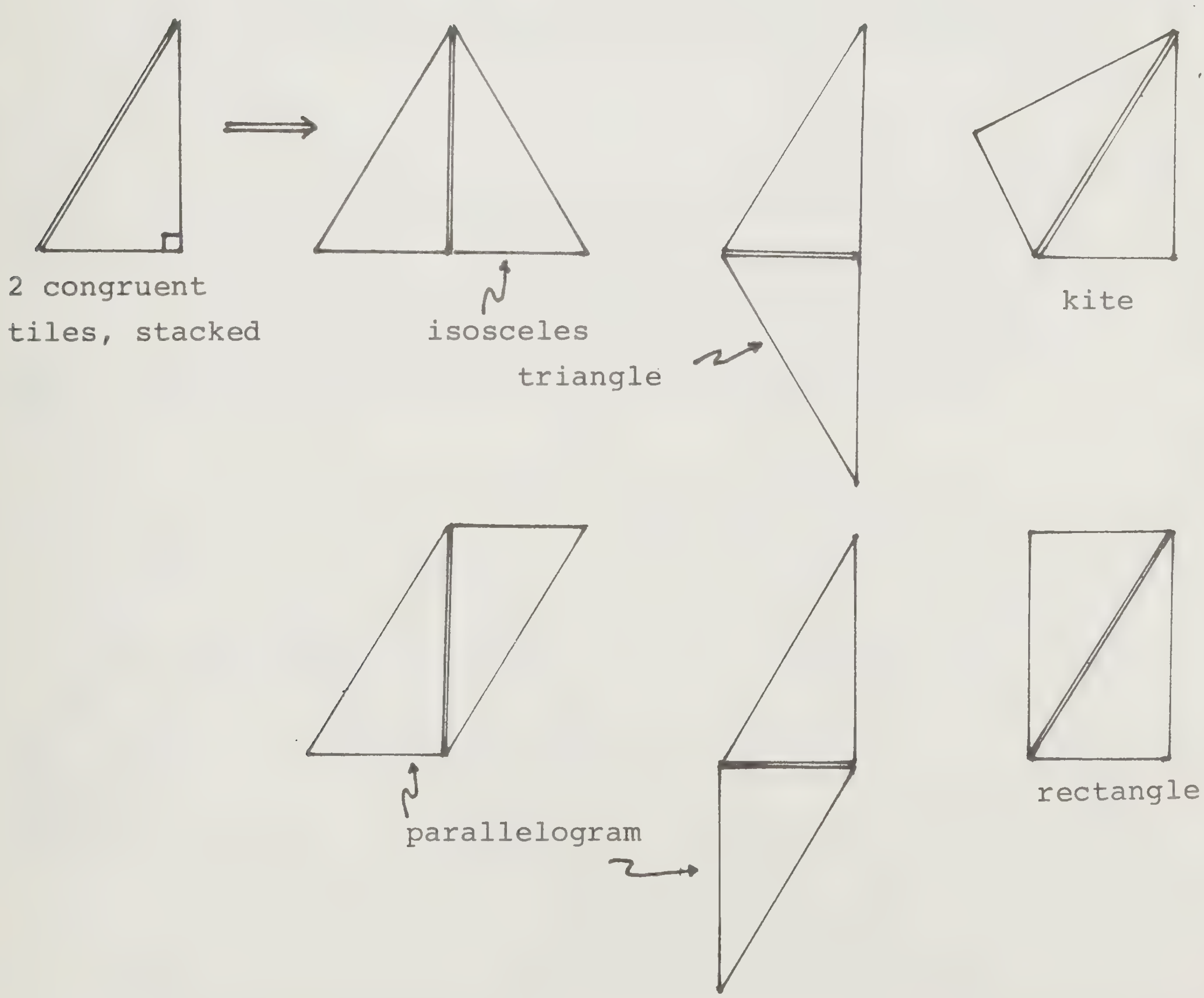
<u>Activity 1</u> (Provide each student with two congruent right-angled triangular tiles.)

i) Stack the tiles, to show they are congruent.

ii) Fit them edge to edge to form new figures, in as many ways as possible.

iii) Make diagrams to represent your results.

(The diagrams below show all possible results; the labelling indicates the names of the corresponding mathematical figures.)



Two right-angled triangles fit together in six ways (as shown above) and make six different figures - 2 isosceles triangles, 2 parallelograms, 1 rectangle, and 1 kite. This activity is useful in introducing or reinforcing the concepts associated with these figures.

<u>Activity 2</u> (Provide each student with 2 congruent right-angled triangular tiles, as in Activity 1.)

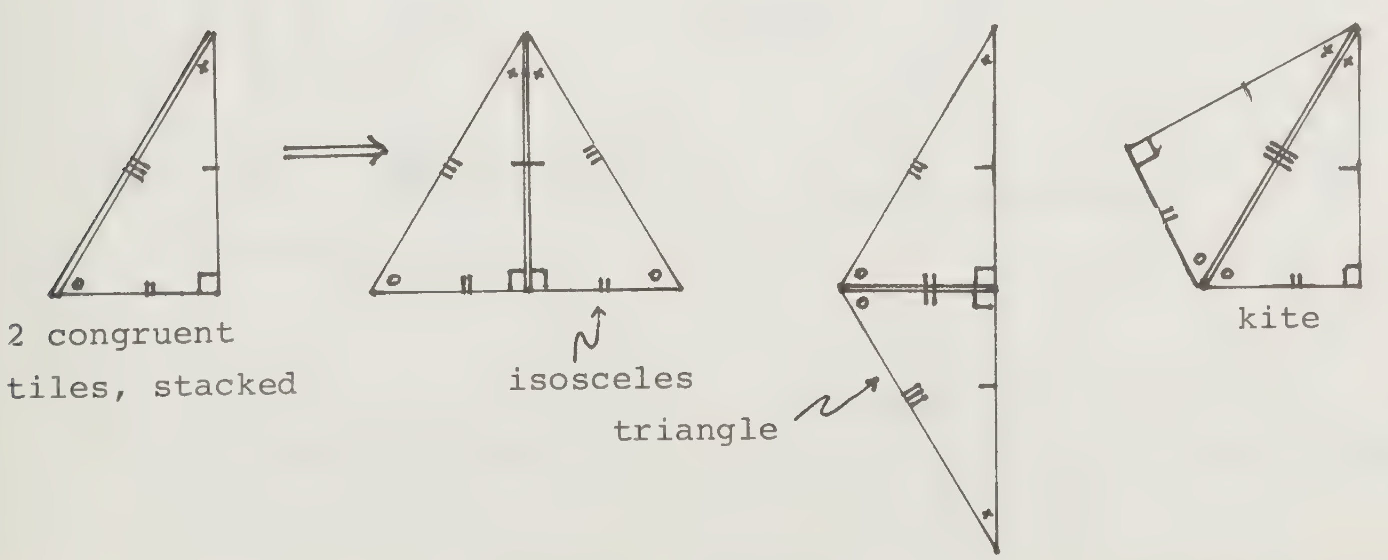
i) Mark the corresponding sides and angles of each of the tiles, on both sides.

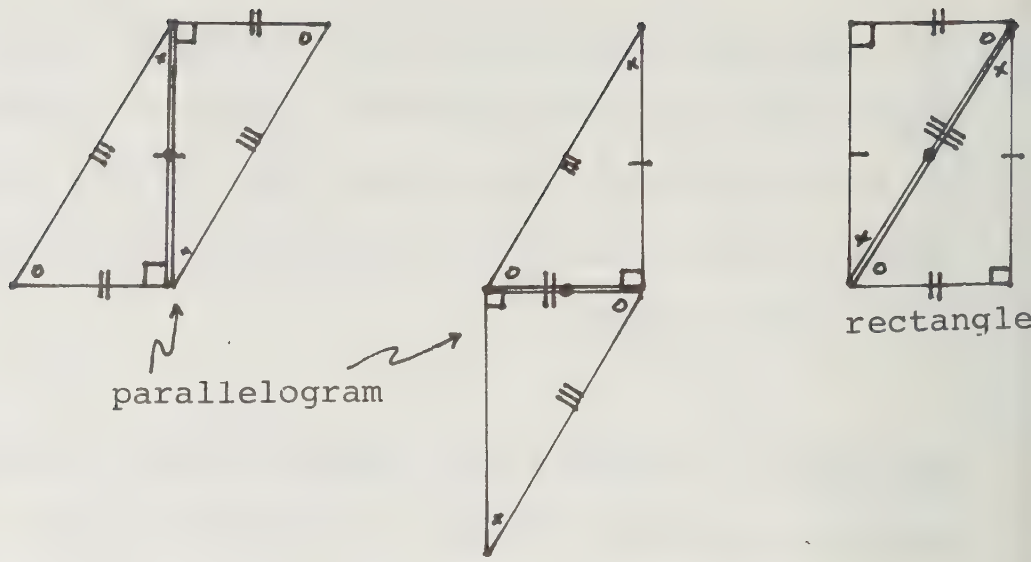
ii) Repeat steps i), ii), and iii) of Activity 1.

iii) Label the equal sides and equal angles on each diagram.

iv) Describe the properties of each figure in words, in symbols.

This activity is related to topics 7G 1d) and 8G 3a). The diagrams below show the results of fitting the tiles. The students' diagrams should be the simplified mathematical representations (i.e. without the double common edges).





Activity 3 (Provide each student with 4 congruent right-angled triangular tiles, as in Activity 1.)

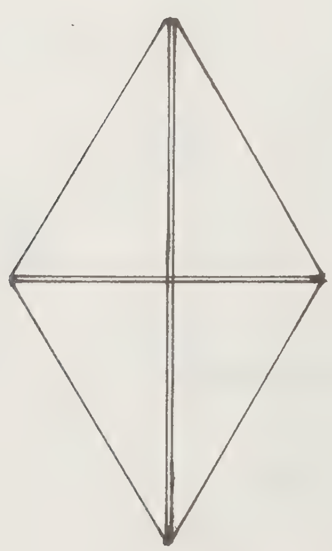
i) Fit the tiles together to form a rhombus, then an isosceles trapezoid.

ii) Make diagrams to represent these figures, including the common edges.

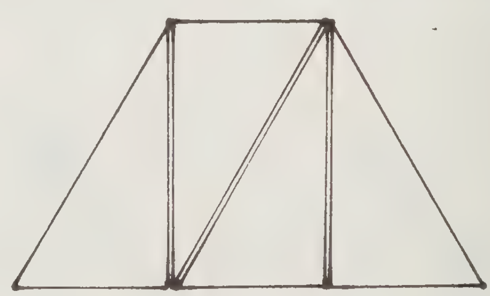
iii) Mark the corresponding sides and angles.

iv) Describe the side-angle properties of each figure.

This activity is related to topics 7G 1d) and 8G 3a).



rhombus



isosceles trapezoid

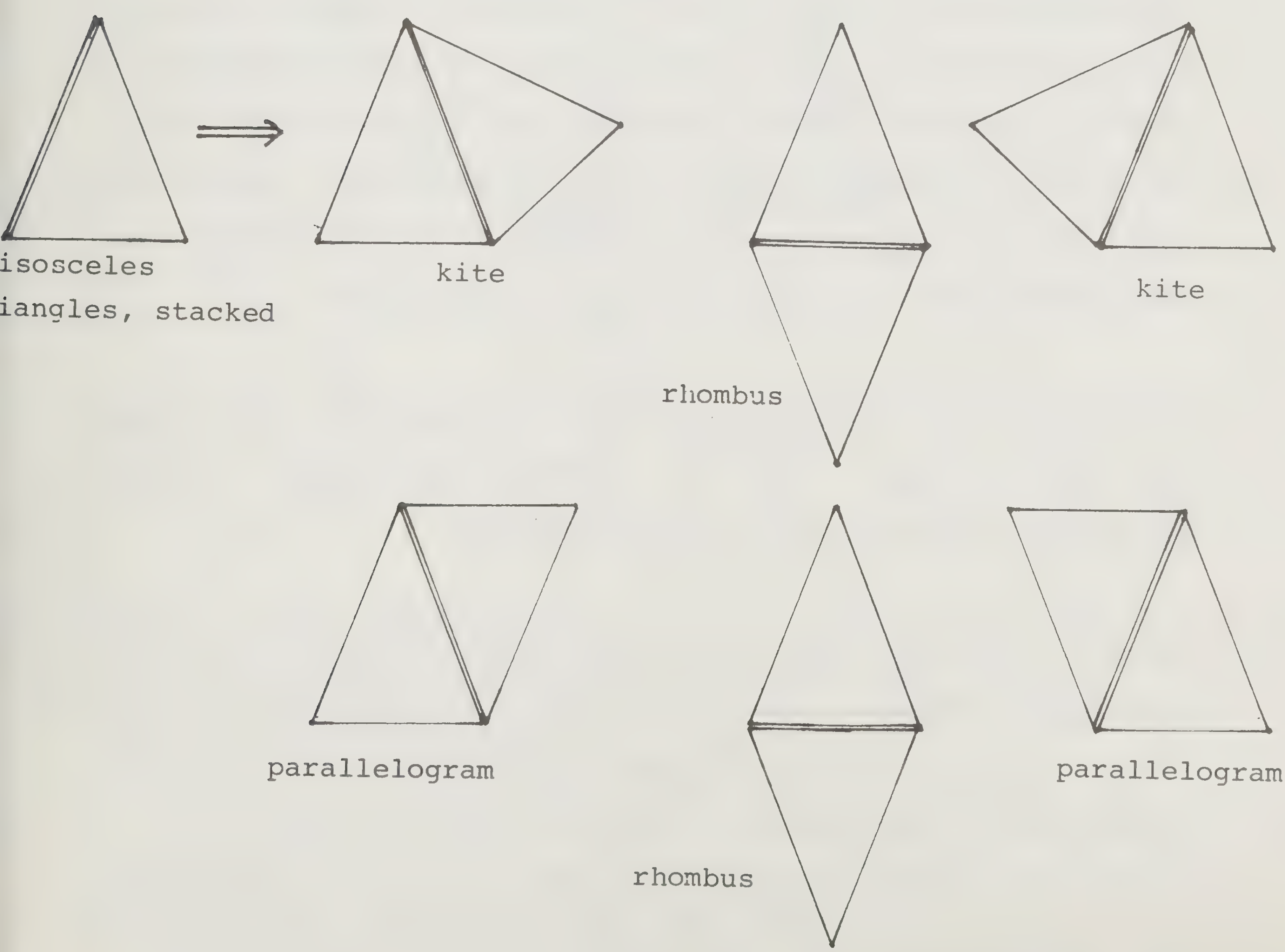
<u>Activity 4</u> Plan this activity similarly to Activities 1 and 2; this time using two congruent isosceles triangular tiles. These will fit together in six ways (as shown below), but in this case the six figures are not all different.

Which figures are the same? This can be determined by

i) moving one of the figures (pair of tiles) until its edges are parallel to the edges of the first figure, or

ii) using the student's diagram, making a tracing of one of the figures and trying to fit it onto the other figure.

The figures formed by the tiles are labelled below. Their properties can be found by marking the corresponding sides and angles.



Activity 5 Plan this activity similarly to Activities 1 and 2, using i) scale triangular tiles ii) equilateral triangular tiles.

The scalene tiles form three different kites and three different parallelograms. The equilateral tiles, although they fit in six ways, produce only one figure - a rhombus.

Activity 6 The above activities can be extended

i) by fitting more than two triangular tiles -- try at least up to six tiles, and

ii) by fitting two or more quadrilateral tiles -- using different types of quadrilaterals.

The purpose of i) and ii) is to determine how many different figures can be formed with a given number of tiles. To do this, the student must compare pairs of figures to determine whether they have the same shape. As the number of tiles increases, the number of possible figures increases and it becomes more of a challenge to distinguish which ones are different. The student then needs to develop strategies to ensure that all cases have been tested and that repetitions have not occurred.

If students work in groups, not all will agree that two figures have the same shape. They will challenge each other. Tracing paper can be used to test each conjecture.

Students should be asked to make diagrams to represent each new figure. These can be cut out and posted on the bulletin board. If the same scale is used, new figures can be tested on the ones already posted. These class projects need not involve much classroom time - once the activity is introduced many students will work voluntarily on their own outside regular classroom time.

Activity 7 (Provide each student with a number of sets of four congruent quadrilateral tiles -- squares, rectangles, rhombi parallelograms, etc., scalene quadrilaterals.)

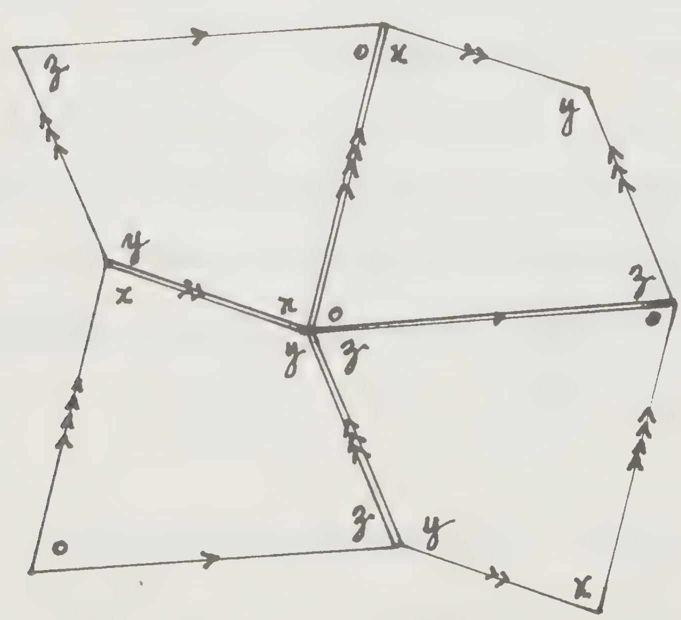
i) Stack the tiles to show they are congruent.

ii) Fit the four tiles edge to edge with a common vertex.

iii) Make a diagram to represent the figure.

iv) Mark corresponding edges, corresponding angles, and parallel lines.

Students will see evidence that the sum of the angles of a quadrilateral is 360°. Also they will see four triples of parallel lines. These properties are shown in the diagram below.



Activity 8 Plan an investigation similar to Activity 7 using six scalene triangles. This will demonstrate that

i) the sum of the angles of a triangle is 180°;

ii) each exterior angle equals the sum of the interior and opposite angles;

iii) there are 3 triples of parallel lines.

Activities such as the above of fitting tiles and of comparing shapes will help students to develop their spatial perceptions. Topic b) provides a more systematic way of doing the above activities.

b) Relating turns, flips, and slides to the figures in topic a)

In the activities that are suggested in topic a), the students are given complete freedom in fitting the tiles edge to edge. Likely, the approach involves random fittings, and so some of the possible figures may be missed. If the decision is made to relate turns, flips, and slides to the activities in topic a), it is suggested that students still be left to explore these activities on their own and to report their findings. At this time, the use of the motions to make the investigations more systematic can be introduced. Some of the students may instinctively use the motions in fitting the tiles. Others will need help in systematizing the approach in the manner indicated by the notes for this topic.

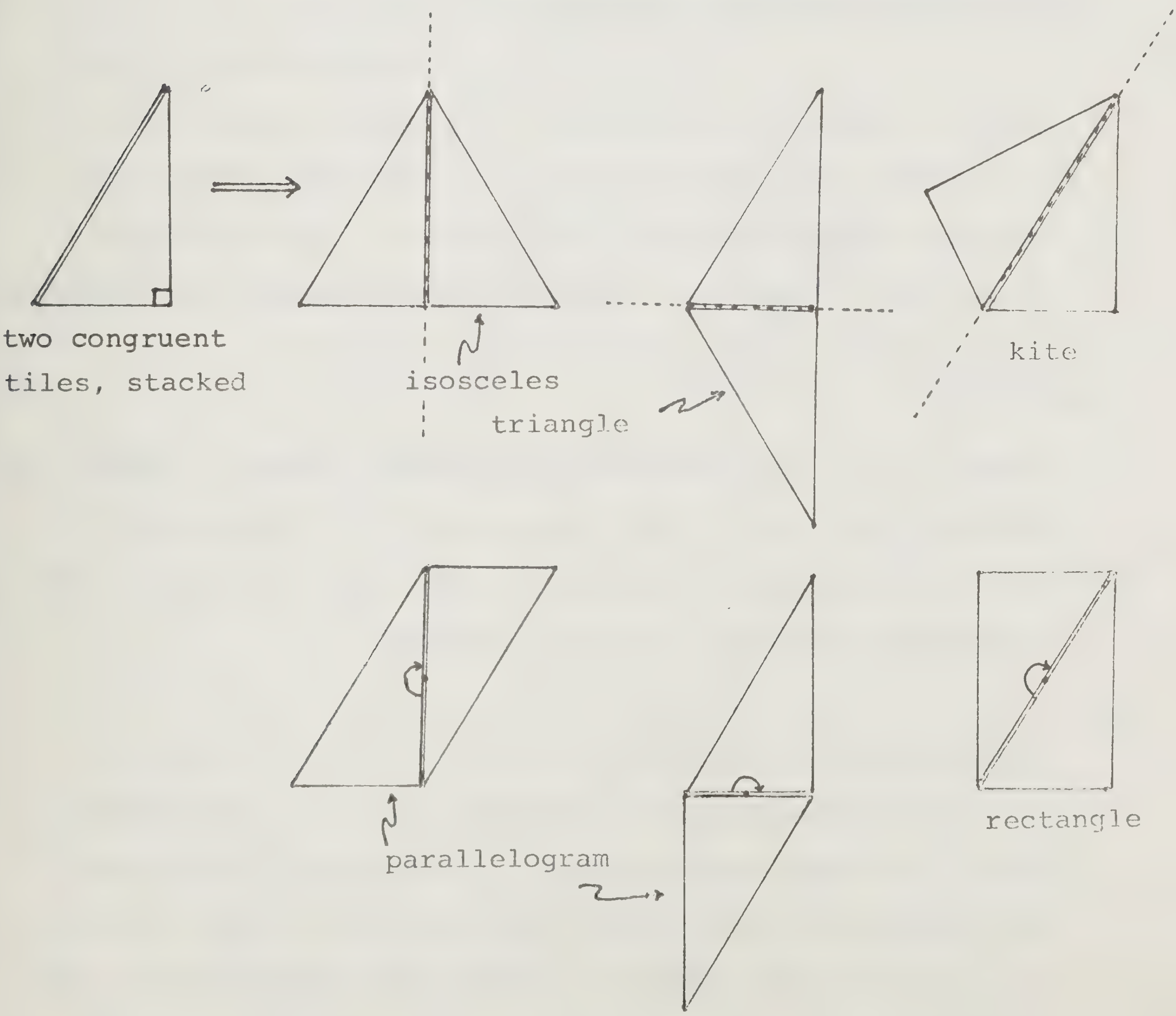
In the following notes, the numbering refers back to the corresponding activities in topic a). The comments show how the motions are related to the earlier activities.

1. Refer to Activity 1 on page 4. Each tile has three edges. The tiles can be fitted edge to edge by

   i) flipping the top tile over each edge (this changes the sense - clockwise, counter-clockwise),

   ii) making a half-turn of the top tile about the mid point of each edge (this preserves the sense).

   Method i) produces two isosceles triangles and a kite — each has a line of symmetry along the common edge. Method ii) produces two parallelograms and a rectangle — each has half-turn symmetry about the mid point of the common edge. See the diagrams below.



When the 'flip half-turn' method is used to generate the new figures, it is not necessary to search out the sides and angles that appear to correspond. The correspondences can be identified by visually (or physically) retracing the flip or half-turn that generated the new figure. This should be done for each figure in a systematic way in order to consolidate this procedure for later work and to prepare for deductive reasoning.

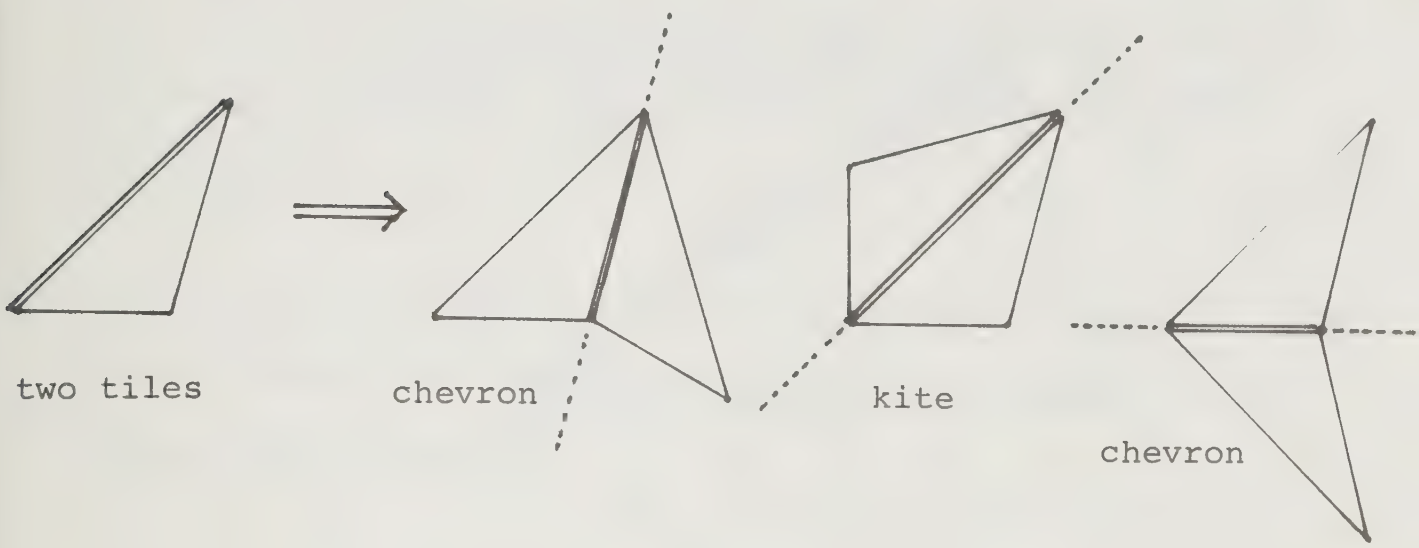
2. Refer to Activity 2 on page 5. Given a figure in which sides and angles are marked congruent, the student should be able to determine whether the figure is symmetrical and to identify the type of symmetry.

3. Refer to Activity 3 on page 6. The student should be able to identify the flips and turns by which the rhombus and isosceles trapezoid have been generated from the stack of four tiles. Then the side-angle properties of these figures can be established by retracing the motions.

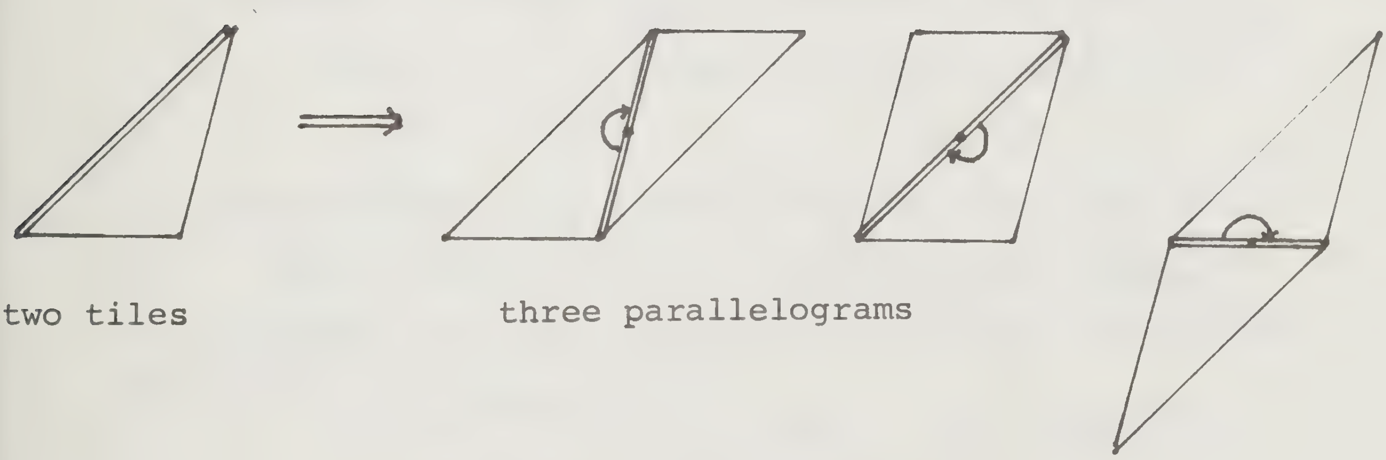
4. Refer to Activity 4 on page 7. The same comments apply here as in 1 above. Some discussion is in order as to why the procedure of Activity 1, but using isosceles tiles, generates only three different figures.

   The tiles have line-symmetry, thus a flip over either of the equal sides produces the same figure (turn one figure until it looks like the other). A half-turn about either of the equal sides produces the same figure (turn one until it looks like the other). A flip over the base and a half-turn about the mid point of the base produce the same figure.

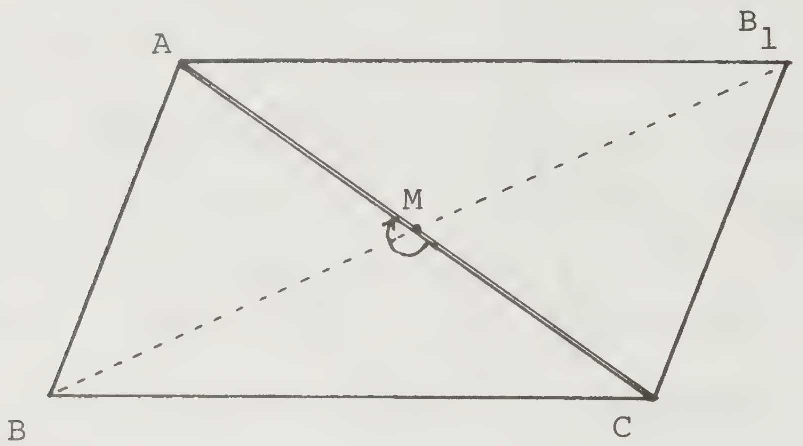
5. Refer to Activity 5 i) on page 8. The flipping of a scalene triangular tile over each side generates three kites (two of which are chevrons, because the tile is obtuse angled)



and the half-turn about the mid point of each side generates three parallelograms.



It is worth noting that any triangle and its half-turn-image about the mid point of a side always produces a parallelogram. One example is shown below.



This example illustrates that under a <u>half-turn</u>:

- a segment and its image are parallel;

  $BC \parallel B_1A$ and $AB \parallel CB_1$

- if the turn-centre is the mid-point of a segment, the segment maps onto itself;

  $AC \longrightarrow CA$

- if the turn-centre is the end of a segment, the segment and its image lie in a straight line;

  BB_1 is a line segment, M is its mid point

- a segment and its image are congruent;

  $BC \cong B_1A$; $BA \cong B_1C$; $MB \cong MB_1$

- an angle and its image are congruent;

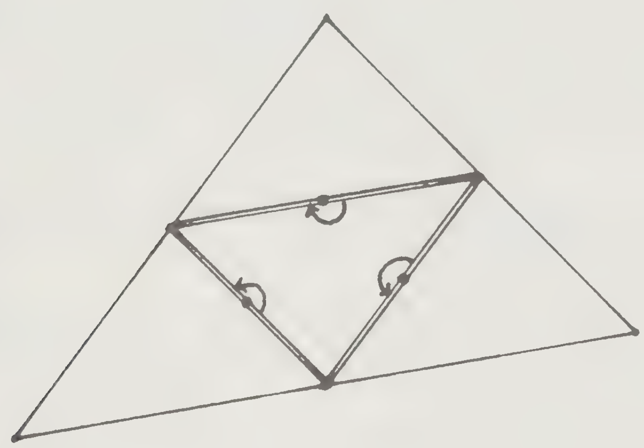
  $\angle ABC \cong \angle CB_1A$; $\angle BAC \cong \angle B_1CA$;

  $\angle BCA \cong \angle B_1AC$

Each of the above properties should be illustrated with tracing paper by every student.

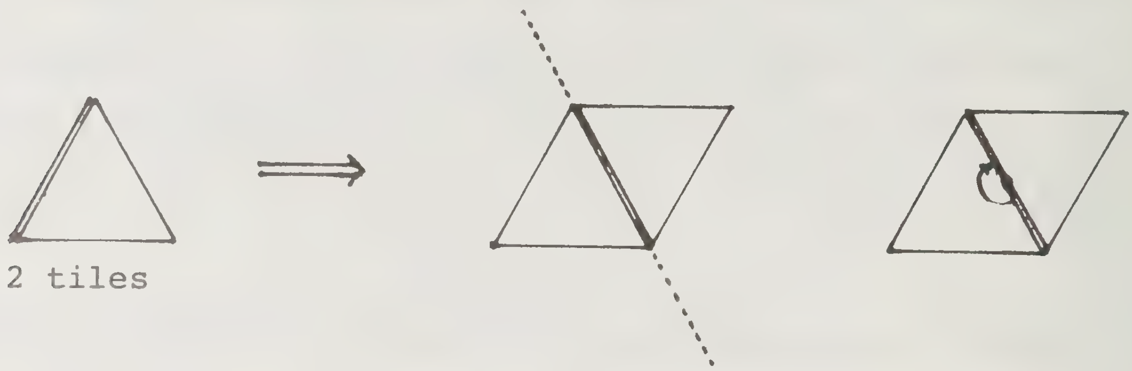
The above comments are intended to indicate the significance of the mathematical ideas that the student is continually exposed to when 'playing' with these tiles. Some of these ideas may be discussed in subtle ways; they should not be treated formally, nor with the symbolism shown above at this time.

If four triangular tiles are stacked, and the top tile half-turned about the mid point of each side in succession, the figure below results.



This illustrates that the segment joining the mid-points of two sides of a triangle is parallel to the third side and equal to one half of it.

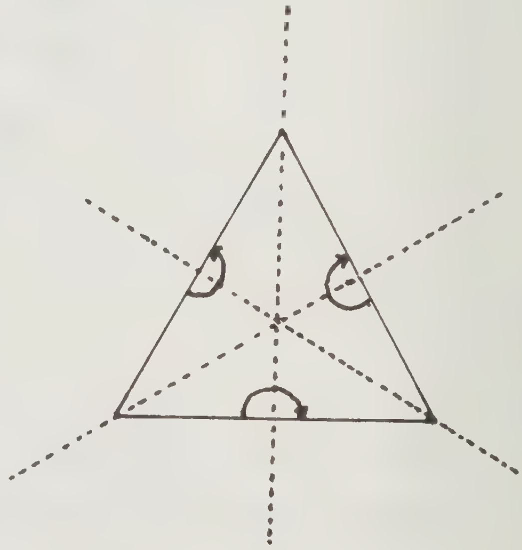
With an equilateral tile, the three flips (one over each edge) and the three half-turns (one about the mid point of each edge) generate only one figure - a rhombus - why is this so?



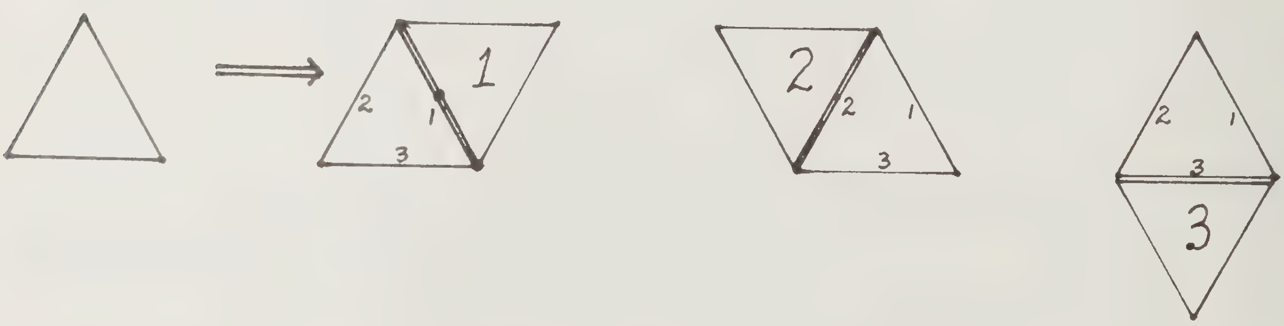
same rhombus by a flip and a turn

6. Refer to Activity 6 on page 8. A systematic approach will help to ensure that all possible fittings have been tried.

i) Consider the equilateral tiles. Each tile has three lines of symmetry and three rotational symmetries. No new situations will occur if the tile is flipped. Students should explain why.

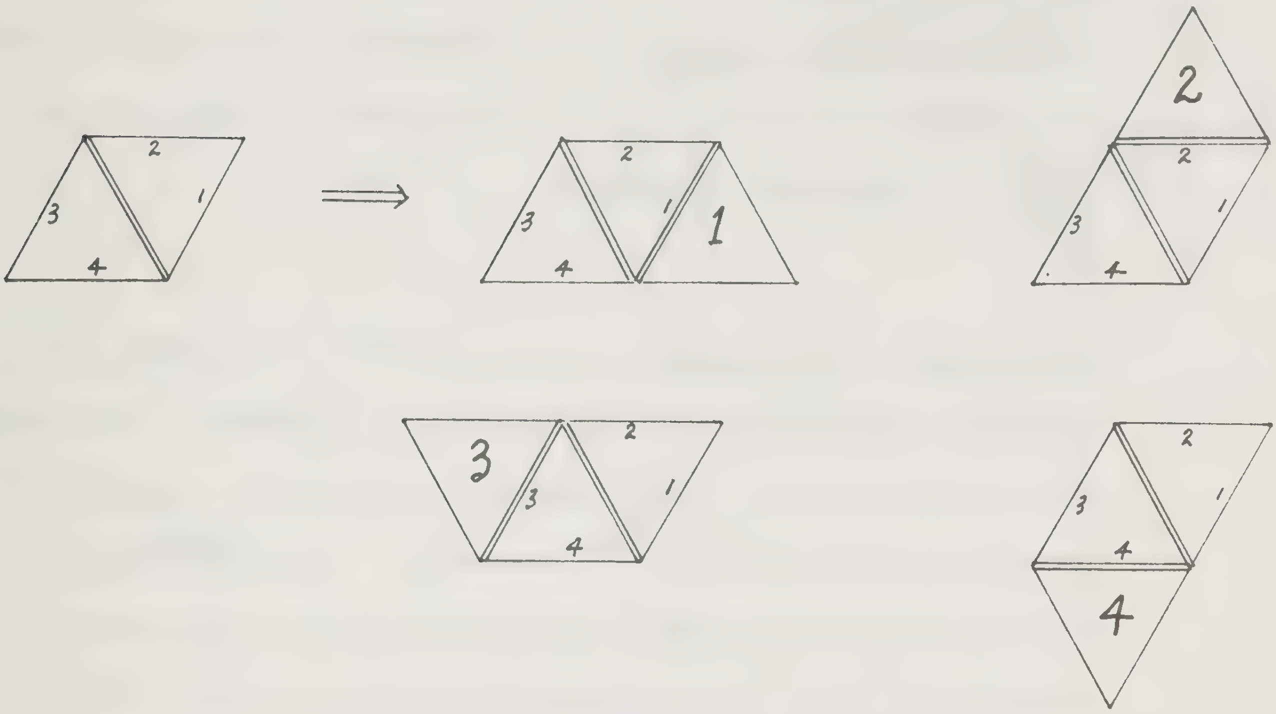


Fit a second tile to the first one in as many ways as possible. The three edges produce three fittings, as shown.



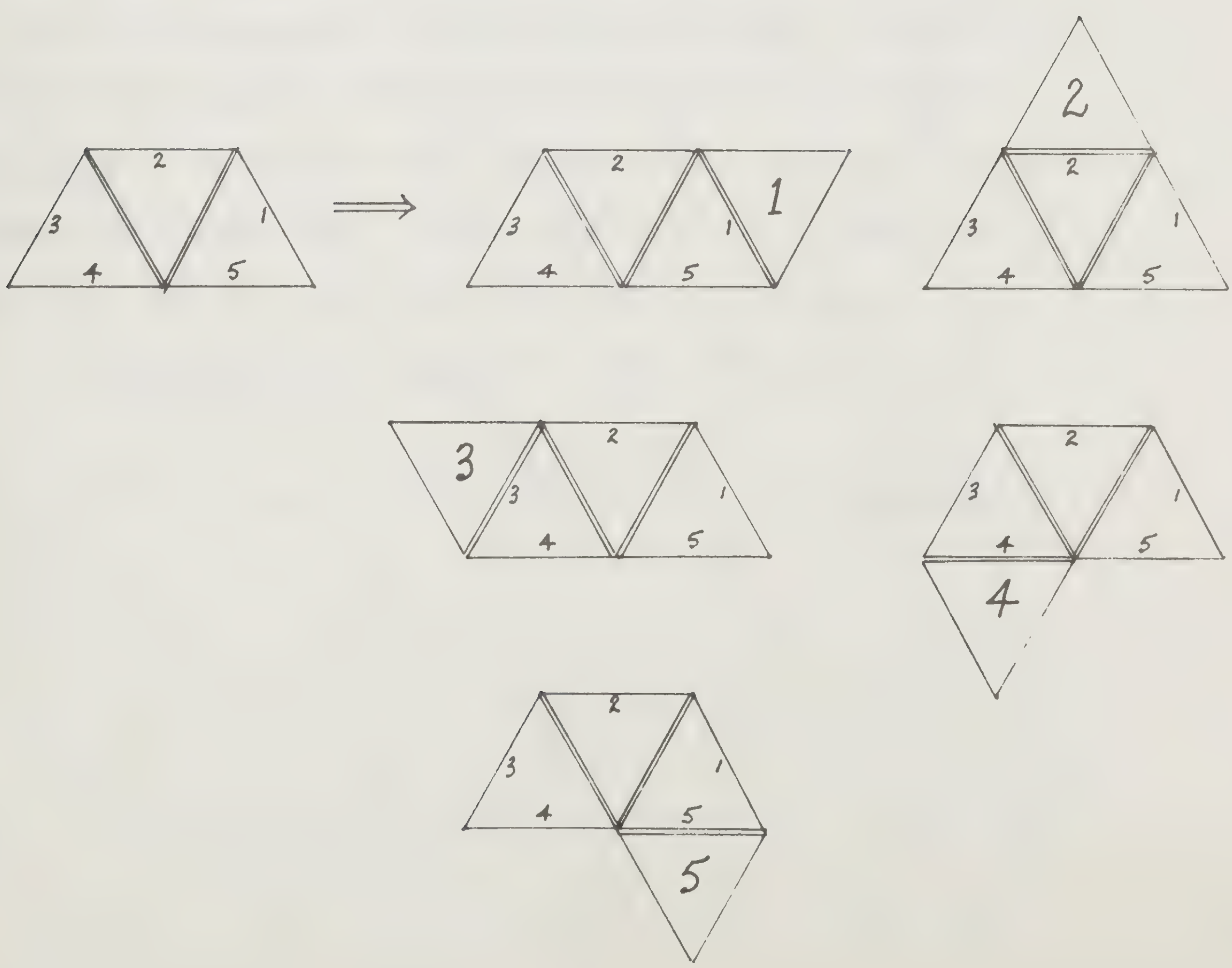
Are the figures the same? If in doubt, use the tracing paper test.

Only one figure can be formed by two equilateral tiles. Fit a third tile on it in as many ways as possible. There are four edges, hence four fittings, as shown.



These figures are all the same.

Continue this strategy.



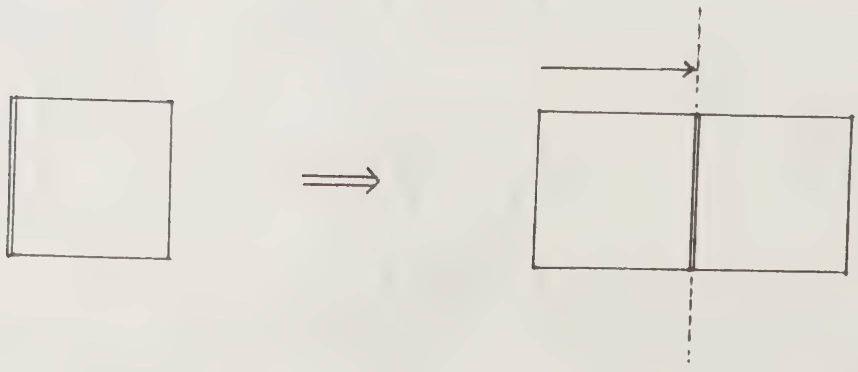
How many different figures are there?

Three different figures can be made from four tiles. Now fit a tile on each face of these three figures. (For five tiles there are 18 fittings but only 4 different figures, for six tiles there are 27 fittings but only 12 different figures.)

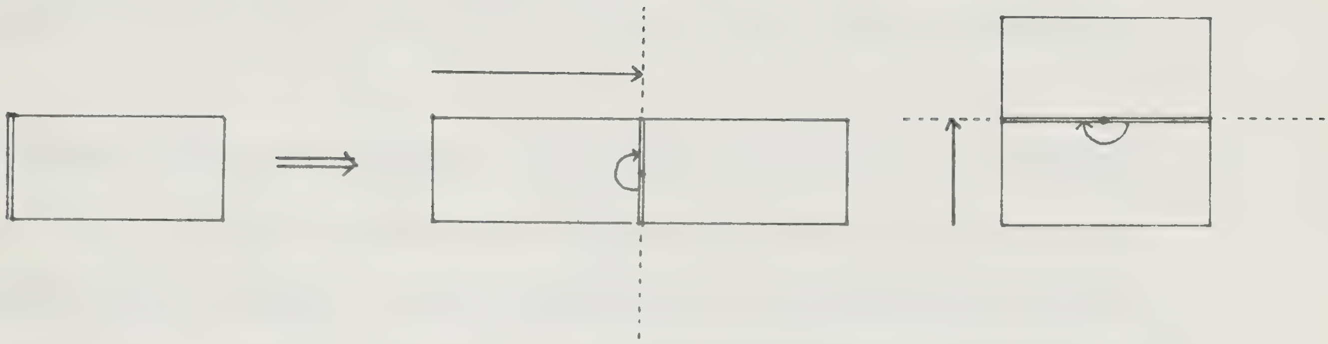
Equilateral grid paper or equilateral dot paper simplifies the drawing of diagrams to represent the above. To be certain that the same figure is not considered twice and to settle all 'arguments', tracing paper can be used to compare the shapes of the diagrams in question. The tracing paper will have to be flipped turned, and slid when making these tests.

ii) Refer to Activity 5 ii) on page 8. Squares, rectangles, rhombi, and parallelograms are illustrated below. They can be fitted by half-turns, flips, and slides as illustrated. In each of these examples, a <u>slide</u> can be used to fit the two tiles. What condition makes this possible? (The opposite sides of the tile must be parallel for this to happen.)

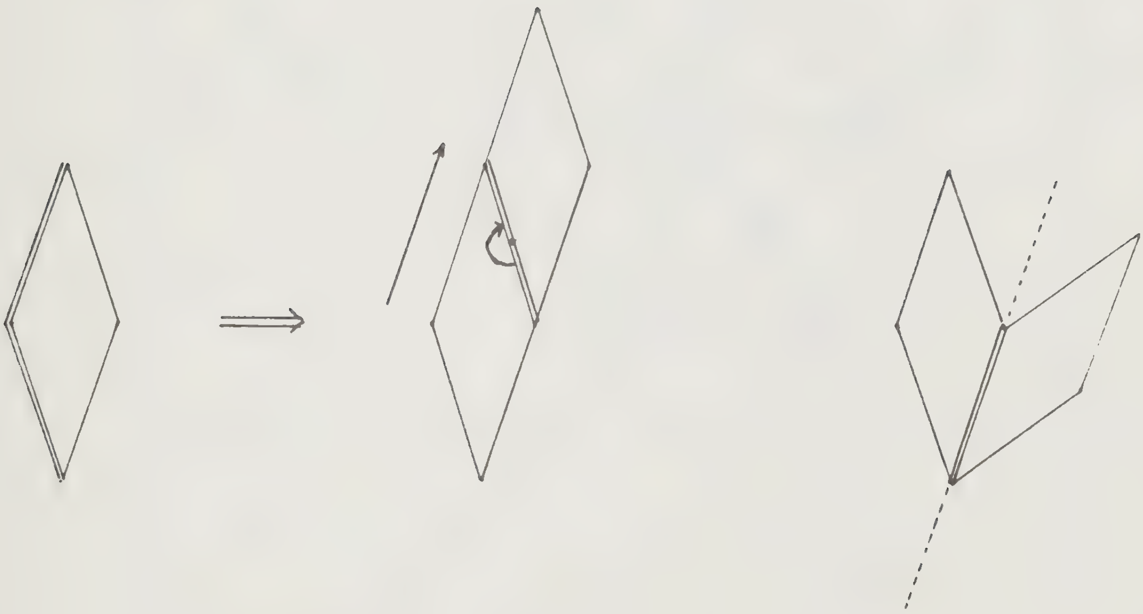
<u>Squares</u>



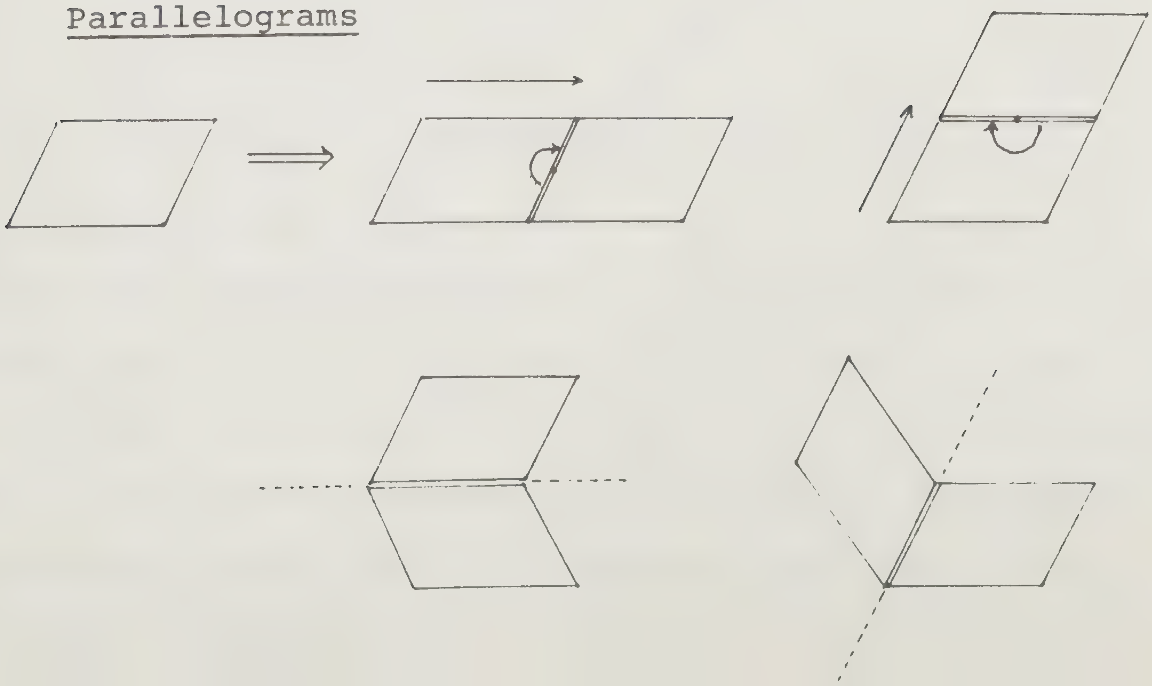
Rectangles



Rhombi

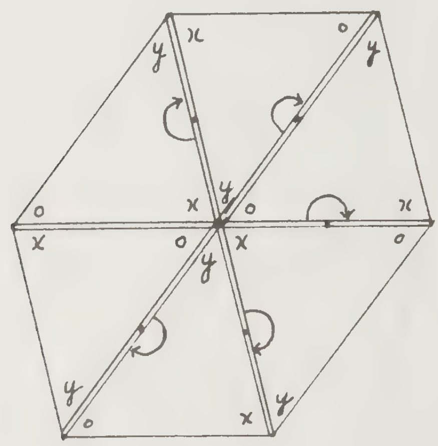


Parallelograms



7. Refer to Activity 7 on page 9. This activity can be carried out systematically by making three successive half-turns about the mid point of an edge. (In this case, move the top three triangles on the first turn, then two, and finally one.)

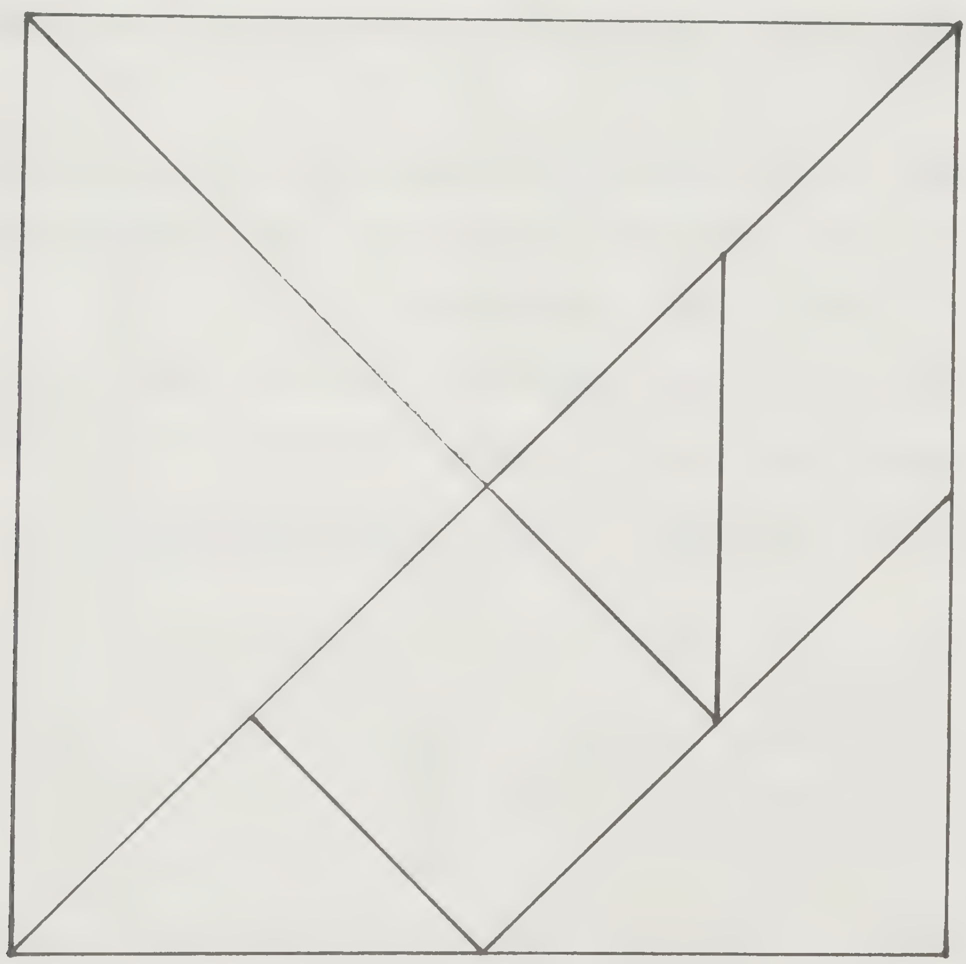
8. Refer to Activity 8 on page 9. In this case five half-turns generate the figure below in a systematic manner. The angle properties tested in Activity 8 occur on six different occasions in this diagram.



c) Tangram and polyominoe puzzles, the symmetries of the tiles and of the figures

Tangrams

The figure below is called a tangram. When it is printed on cover board and the individual figures are cut out, seven tangram tiles result. These tiles can be fitted i) edge to edge and ii) in other ways to form a variety of figures. Each of these new figures is also called a tangram.



For example, students may be asked to make a rectangle, parallelogram, isosceles trapezoid, right-angled isosceles triangle, right-angled trapezoid. There is almost an endless collection of figures that can be made from the tangram pieces - other geometric figures, caricatures, and representations of real world objects. These are not described here, since there are many sources of problems in the literature.

Tangrams are challenging to many students. Once the basic idea is introduced along with a source of puzzles, many students will spend hours outside the classroom 'playing' with these puzzles. These experiences will help to sharpen spatial perceptions and encourage creativity. The manipulation of the tiles provides intuitive experiences with slides, turns, and flips.

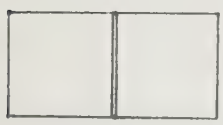
There are seven tiles, but only five different figures, and of these only three shapes -- right-angled isosceles triangles of three sizes, a square, and a parallelogram. The following ideas are significant in developing strategies for solving the puzzles.

- Only one of these figures does not have line-symmetry. Which one? Only it needs to be turned over to form a new fit. Why is this significant?
- How are the five triangles related? How is this significant?
- How are the small triangles related to the square, to the parallelogram? How is this significant?

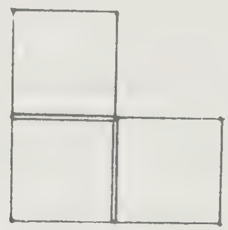
Most major school suppliers list 'tangram books' in their catalogues, each containing a great variety of puzzles. In addition, many textbooks refer to tangrams and many articles have been written on them in professional journals. There seems to be no shortage of tangram puzzles. Because of the abundance of references, the notes on tangrams are brief.

Polyominoes

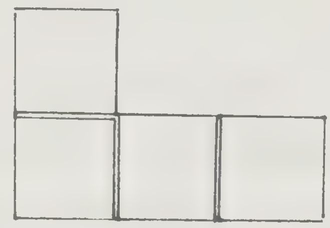
A polyominoe is a figure formed by 2 or more squares placed edge to edge. The following are examples of polyominoes of 2, 3, 4, 5, and 6 sides.



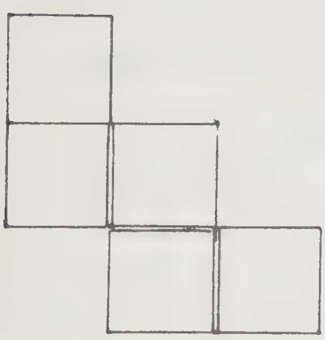
dominoe



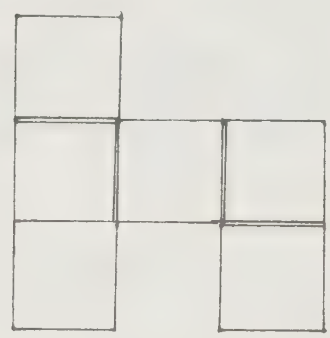
triominoe



tetronimoe



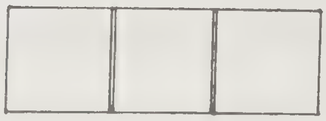
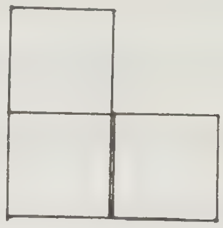
pentaminoe



hexaminoe

Initially, students may be encouraged to search for how many there are of each type -- by constructing them with tiles, then drawing pictures of them on dot paper. This activity is similar to the one described in the notes for topics 3a) and b) involving equilateral tiles. As the number of squares gets larger, the need for a strategy increases. In order to compare the shapes of two polyominoes, it is necessary to consider whether they are turn-or flip-images of each other - either visually, or with the use of tracing paper.

There are only two different triominoes.

(i)  (ii) 

To find how many different tetrominoes there are, consider all positions for fitting a square on each of i) and ii). There are 8 positions for i), 7 for ii) - in all, 15 possible figures. Only five are different. Some students can make these decisions mentally, others will need to draw all fifteen figures and then, by visual flips or turns, or by using tracing paper, decide those that are the same.

This method can be extended to find all possible pentaminoes, and hexaminoes.

Pentaminoes are interesting for a number of reasons.

i) There are 12 different pentaminoes. Finding all 12 is a challenge. Use square tiles and record the results on a rectangular grid or dot paper.

ii) The area of each is 5. Draw a rectangular grid 10 by 6 (ruled or dots). Try to fit all 12 pentaminoes into this grid at the same time. (There are thousands of ways; just find one.)

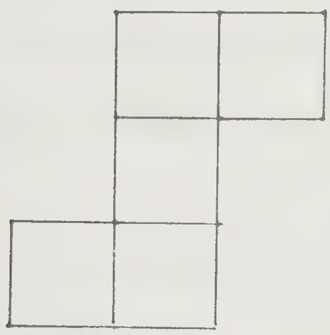
iii) Make a grid 8 by 8. Using a set of the 12 different pentaminoes player A places a pentaminoe in the grid. Then player B, then A, etc. The last player to make a successful move wins. This is a great strategy game involving symmetry, flips, turns, slides and lots of foresight.

iv) Try to fit all 12 pentaminoes on the 8 x 8 grid.

v) What is the least number of pentaminoes that will 'block' the grid? Which pentaminoes are they?

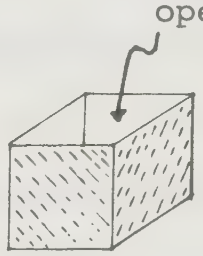
vi) Which pentaminoes can be used repeatedly to tile the plane? (These are called reptiles.)

vii) Which pentaminoes can be used as a net to form an open cube? For example,



pentaminoe net

open cube

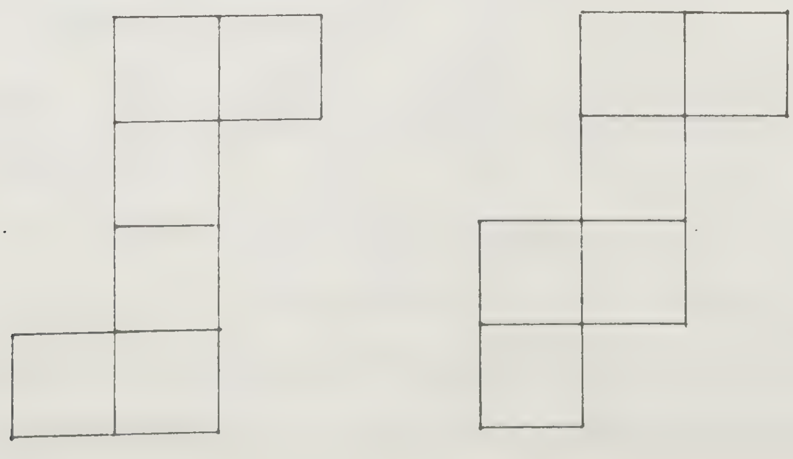


Hexaminoes

How many different hexaminoes are there? This case is much more complex than for pentaminoes, and will challenge the best students. (There are 35.)

Some hexaminoes are nets for a cube. Finding which ones is an interesting exercise - every student will have some degree of success.

hexaminoe nets



d) Tiling the plane

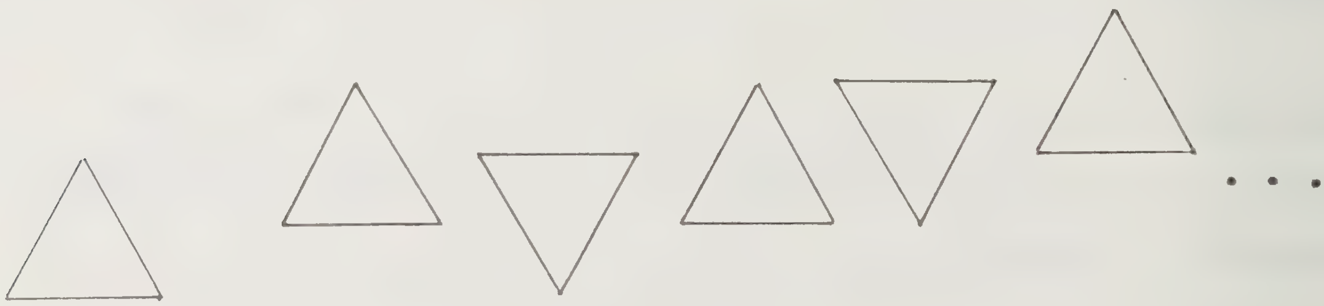
The activities in topics a), b), and c) provide a good introduction to this topic, particularly if a systematic approach is used. However, success has been experienced starting this topic 'cold', then extracting some of the specific information of the previous topics from the tiling pattern.

Tiling the plane, at its simplest level, involves

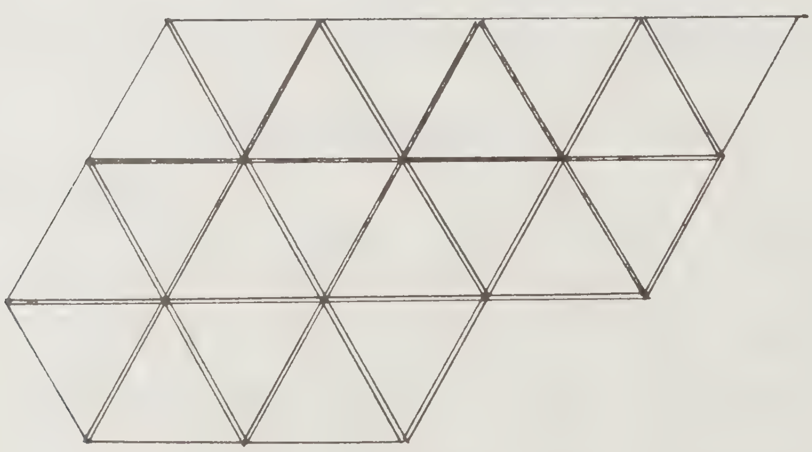
- . using a set of congruent tiles (all of them the same shape and size), and
- . fitting them together so that they touch each other edge to edge without gaps.

Example:

A set of equilateral triangle tiles



will fit together to form a tiling pattern



A sufficient number of tiles should be used to show that the pattern can be continued endlessly in all directions. When

this is possible, we say that we can tile the plane with the given figure.

Students should be supplied with different sets of congruent tiles and asked to see if they fit to 'tile the plane'; then asked, "In how many ways?"

The tiles can be cut from cardboard or other stiff material. Analysing patterns is easier if the material used has a contrasting colour on the reverse side. (See the notes for 7G 3f).)

Students may work in groups and should make diagrams to represent the patterns they generate. They should discuss their patterns with each other. The patterns can be coloured in different ways to highlight different features.

The plane can be tiled with any shape of triangle (equilateral, isosceles, right angled, scalene) and any shape of quadrilateral (square, rectangle, parallelogram, etc.). The most interesting quadrilateral is the general case i.e. using scalene quadrilaterals. It is not at all obvious that this case will work; see the notes for Activity 6 in topics a) and b).

The topic can be extended to examine:

i) other figures, such as regular polygons (only the hexagon works), or polyominoes;

ii) Escher type figures (see the reference below - the art patterns, the strategy for building these special tiles);

iii) tilings with 2 or more basic figures;

iv) Altair paper designs (see Longmans Publishing Company of Canada);

v) the uses of tiling patterns in the art forms of all stages of civilization and in modern designs (tiled floors, vinyl floor covering, wallpaper, fabrics);

vi) stacking of space with 3-D objects with real-life applications (packaging of commercial products, and brick laying, for example);

vii) the special topics of e) and f) that follow.

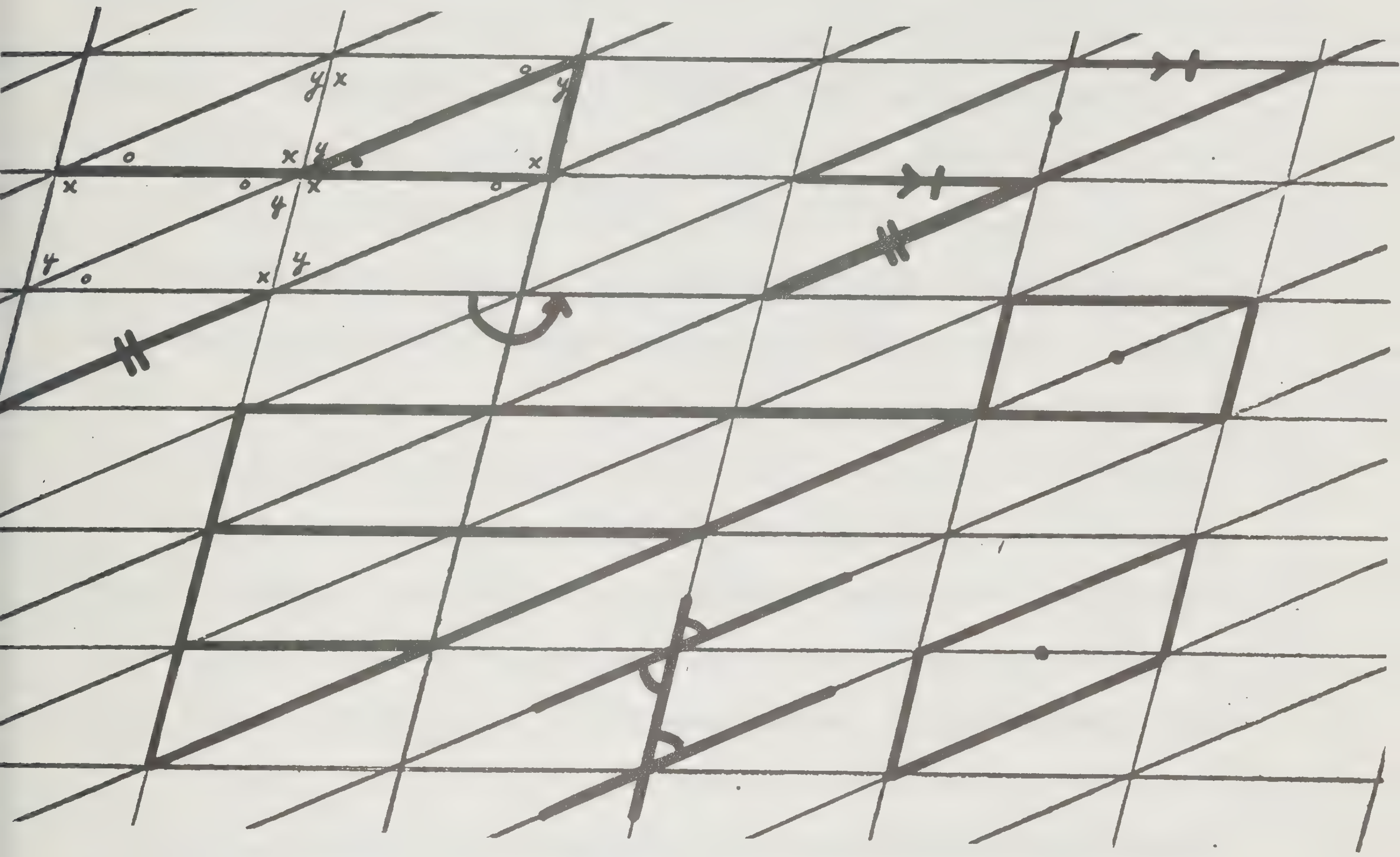
See Chapters 4 and 5 of Geometry, An Investigative Approach; Addison Wesley Publishing Company, as a good reference for many of the above topics.

e) Identifying congruent figures, similar figures, line and angle properties in tiling patterns

Tiling patterns occur frequently in our environment - in art designs and in buildings, to mention only two examples. Students should be encouraged to examine such patterns and, when possible, to bring samples to class (clippings from magazines, wallpaper samples, and student drawn diagrams of these situations). There are many examples of geometric concepts and properties in these designs (parallel lines, perpendicular lines; special triangles, quadrilaterals, polygons; specific cases of traditional theorems of geometry; and the transformations of translation, rotation, reflection, glide-reflection, and dilatation -- see the notes for topics f) and g) that follow).

The following pattern is a tiling of the plane using congruent scalene triangles. It is provided as an example of the geometric properties that are illustrated in tiling patterns. (Corresponding

sides and angles are congruent, the properties are illustrated in infinitely many positions.)



- When two lines intersect, the opposite angles are congruent.
- The sum of the angles of a triangle is 180°.
- Each exterior angle of a triangle equals the sum of the opposite interior angles
- A triangle and its half-turn image about the mid-point of each side form a parallelogram (three different ones).
- The segment joining the mid-points of any two sides of a triangle is parallel to the 3rd side and equal to one half of it.
- The segment joining points that divide two sides of a triangle in the ratio m:n (m,n are whole numbers) is parallel to the 3rd side and equal to $\frac{m}{m+n}$(length of 3rd side).
- When a transversal intersects two parallel lines i) the alternate angles are equal; ii) the corresponding angles are equal; iii) the sum of the interior angles on the same

side of the transversal is 360°.

. A segment and its half-turn image about a point are equal and parallel.

. Opposite sides of a parallelogram are equal.

. etc.

If a different tiling pattern is used, some new properties emerge. Students should be encouraged to look for properties in the tiles they see from day to day.

It is not intended that all of these properties will be identified, much less formalized at this time. However, many of the properties that are identified in 7G 1,2, and in 8G 2 are illustrated in these patterns.

Another purpose of the above example is to illustrate for teachers the richness of the geometry that can be found in tiling patterns. If students have had experiences in building tiling patterns in Grades 7 or 8, then teachers in secondary schools may use these experiences as a way of introducing or reinforcing many geometric properties from specific cases. Class sets of patterns for various tiles could be prepared for use by individuals or by groups of students. For example, a scalene triangle tile can be used to generate four different patterns. Students could generate lists of properties, such as on the previous page.

f) Identifying slides, turns, flips and symmetries in tiling patterns

There are two aspects to this topic.

1. The first involves identifying the transformation by which any pair of figures in the pattern are related, and the symmetry of figures that are created by two or more of the tiles.

   i) The basic figures (tiles) are all congruent; thus they are related by one of the transformations: translation, rotation, reflection, or glide-reflection. Any two figures could be selected and the students asked to describe the transformation (or optionally, a combination of transformations) by which the one figure is related to (maps onto) the other. If the transformation is a reflection or a half-turn, then the figure and its image unite to form a symmetric figure.

   ii) In many patterns the shape of the basic figure appears again and again in varying sizes. These figures are related by dilatations. (See the figure below.) The students could be asked to find the centre of dilatation and determine the scale factor for any two such figures. It can be observed that:

   . corresponding angles are congruent;

   . corresponding areas are in the ratio $k^2:1$;

iii) The term 'tiling the plane' is associated with the act of fitting real tiles to form a pattern and the diagram that is made to represent the pattern.

The same pattern can also be generated by using only one tile; tracing it; then moving it by a slide, a half-turn, or a flip to a new position and tracing it; and repeating the process many times until it is apparent that a tiling pattern is possible. This process has its limitations if an accurate pattern is desired, since it is not possible to trace a figure exactly along an edge. However, this second procedure can be carried out more formally by using accurate construction methods to form the tiling patterns; i.e. the methods that are suggested in the notes for 8G 2a).

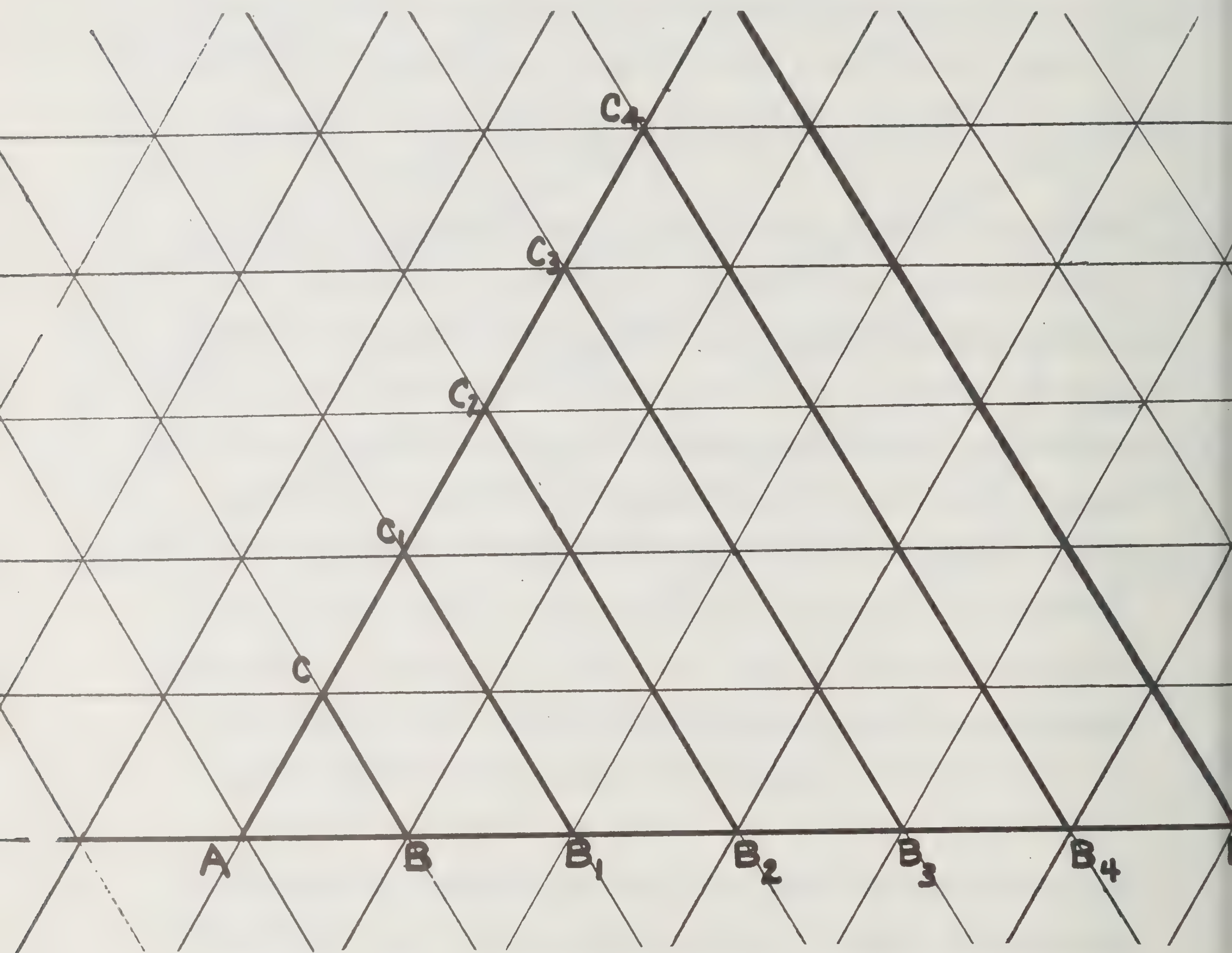
When the pattern is generated by the above method, the process is usually referred to as tesselating the plane, and the result is called a tesselation of the plane. Tesselating the plane involves the application of transformation properties and accurate constructions. These latter remarks are for the benefit of teachers, and are likely of little significance to students at this stage.

2. The second aspect of this topic involves finding the symmetries of infinite tiling patterns. This can be done on the overhead projector. Two acetate copies of a full page tiling pattern will be needed -- one black, one coloured (say red). Sample patterns are provided on the next two pages. (The reproduction process introduces distortion in the copies - these irregularities can be minimized by aligning the copies in the same way.) Place the red copy on the black copy so that they fit. Now move the red copy slowly in a direction to produce another fit. Slide it, or turn it about a point, or flip it -- to make a new fit of the patterns. This process is the same as the tracing paper test that was used to discover the symmetries of a single figure. See the notes for 7G 1c). Each motion identifies a symmetry of the plane. Repeat the process many times to suggest the infinite order of symmetry of the tiling pattern.

Note that the concept of translational symmetry can only be discussed in terms of an infinite tiling pattern. A finite figure cannot fit onto itself by means of translation.

g) Area concepts in tiling patterns

Many tiling patterns show dilatation images of the same figure over and over again. An example is given by the pattern below.



Under a dilatation with centre A and:

scale factor 2 $\triangle ABC \longrightarrow \triangle AB_1C_1$

scale factor 3 $\triangle ABC \longrightarrow \triangle AB_2C_2$

scale factor 4 $\triangle ABC \longrightarrow \triangle AB_3C_3$

scale factor 5 $\triangle ABC \longrightarrow \triangle AB_4C_4$

.
.
.

scale factor k $\triangle ABC \longrightarrow \triangle AB_kC_k$

By considering the area of $\triangle ABC$ to be 1, then the areas of the other triangles are as follows:

$\triangle AB_1C_1 = 4$ $\triangle AB_2C_2 = 9$ $\triangle AB_3C_3 = 16$ $\triangle AB_4C_4 = 25$

and in general, for each value of k the image triangle has area k^2.

The same observation applies to the other triangles in the figure, to the parallelograms, and to any other figures that can be 'highlighted' in the pattern.

The above examples also illustrate that

$B_1C_1 = 2\ BC$

$B_2C_2 = 3\ BC$

$B_3C_3 = 4\ BC$

$B_4C_4 = 5\ BC$

or, in general, that

. the image segment of AB has length k(AB).

Also, it may be observed that

. the size of an angle is unchanged;

. a segment and its image are parallel.

The above comments are significant for topics 8G 4d), 9 Adv. G 4d), and 9 Gen. G 4d).

Ministry of Education Ontario

GRADE 7 GEOMETRY

SECTION 4: TRANSFORMATIONS

RELATED SECTIONS AND TOPICS

PAST	FY:	Pages 7, 12 symmetry, slide, turn, flip, congruency
	Ed PJ Div:	Pages 74 - 76
PRESENT	Gr 7:	A lb; A 3; G l; G 2ab; G 3; G 5a; G 6e
FUTURE	Gr 8:	G la; G 2; G 3abd; G 4abd; G 5df
	Gr 9 Gen:	G lab; G 2b; G 3; G 4; G 5
	Gr 9 Adv:	G la; G 2bef; G 3; G 4; G 5
	Gr 10 Gen:	G lab; G 3
	Gr 10 Adv:	G lab; G 2; G 3; G 4; G 5

a) Image of a figure under a slide, turn, or flip

Slides, turns, and flips are called motions. They are associated with activities in which physical objects (such as tiles, tracings of figures, and blocks) are moved from a first position to a second position, according to a given rule. The study of these motions and of their properties and applications is popularly called motion geometry. Activities involving geoboards, mirrors, paper folding, drawings on dot paper (or on grids), or pattern blocks, are a part of the study of motion geometry.

The distinction between these motions and the transformations of translation, rotation, and reflection is subtle, and is discussed in the notes for topic b).

Ideas related to slide, turn, and flip have been introduced in the mathematic program of the Junior Division (The Formative Years, page 12) along with related concepts of the symmetries of figures in the plane and of objects in space. In planning this section of the Grade 7 course, teachers should determine the extent to which these topics have been developed in earlier years, including the nature of the activities. They should build upon these experiences, avoiding unnecessary repetition.

There are numerous examples of physical objects in the environment that move by slides, turns, and flips, or by combinations of these. The transition from real-world motions to geometry is made when diagrams are drawn that represent the 'before' and 'after' aspects of the motions or of mirror reflections. These diagrams are mathematical models of real world situations.

Creating models and working with them is a specific expectation of the aims for the mathematics program of the Intermediate Division (see pages 1 and 13 of Intermediate Division Mathematics 1977, Draft Copy).

There have been frequent references to slides, turns, and flips in the notes for sections 1, 2, and 3. Ideas of motions pervade the other topics of geometry and, ideally, should be integrated into their development. If the motions are treated in this way, a separate development of topic a) will not be necessary.

There is a fourth motion, called glide-reflection (slide-flip), that is not referred to in The Formative Years or the Grade 7 and 8 program. It is introduced in the program for Grades 9 and 10. Many of the commercial reference materials on motion geometry discuss this motion. The most common example of a glide-reflection is the pattern formed by 'footprints in the sand'.

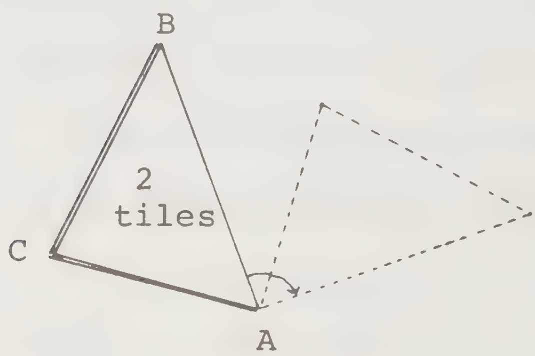
The following list suggests a variety of activities. In making selections, teachers should avoid unnecessary repetition of earlier experiences. In this study, references may be made back to the uses of slides, flips, and turns as described in the notes of earlier sections.

1. Activities with two congruent tiles

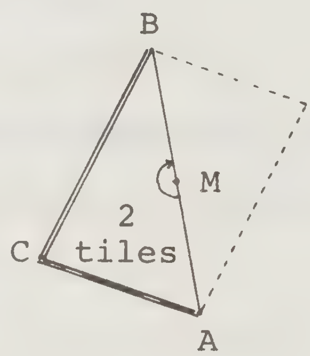
In these activities, the upper tile is

i) turned about a point (vertex of the tile, mid point of an edge, more general point);

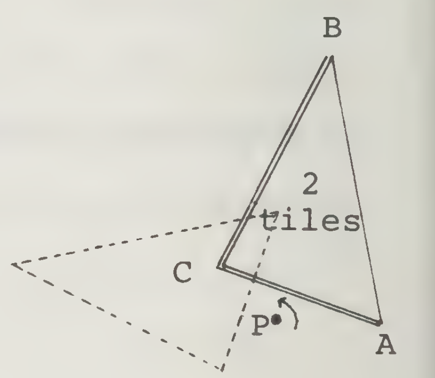
90° turn about A
(clockwise)



180° turn about M

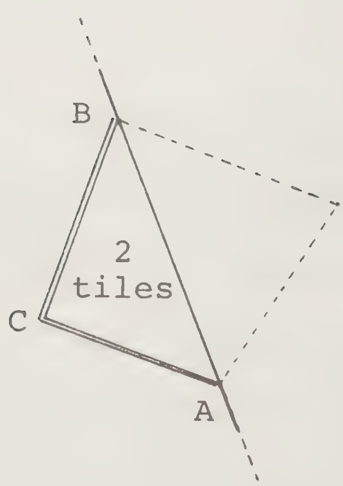


180° turn about P

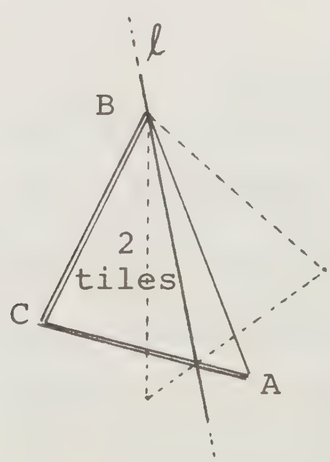


ii) flipped over a line (the edge of the figure, within the figure, outside the figure);

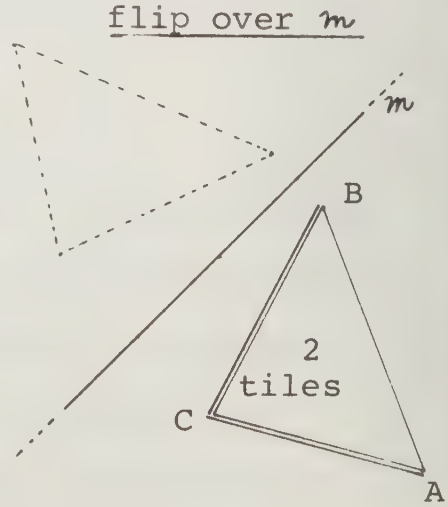
flip over AB



flip over ℓ

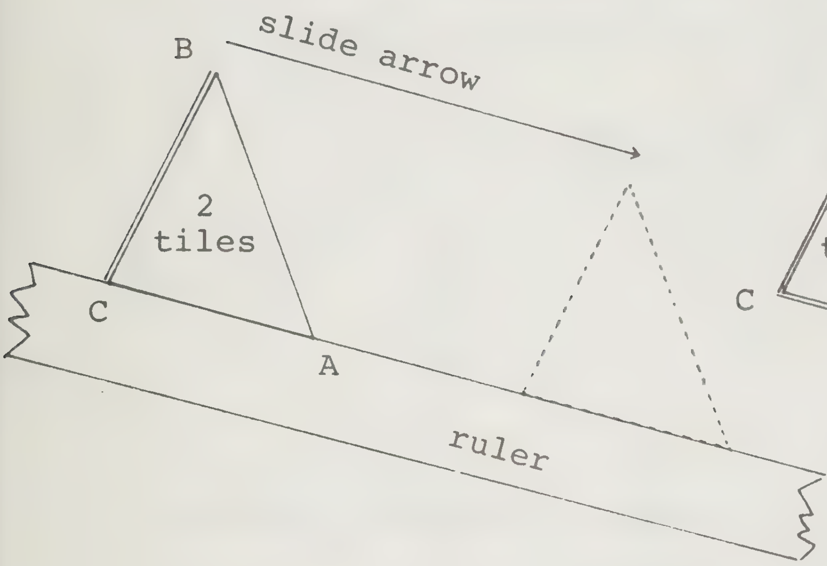


flip over m

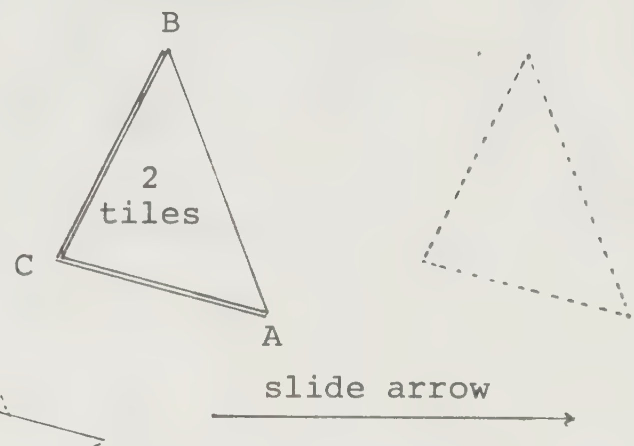


iii) <u>slid</u> a given distance in a given direction (slide arrow parallel to an edge - slide the edge along a ruler, slide arrow in any direction - draw parallel 'tracks' through the vertices of the figure).

<u>slide parallel to CA</u>

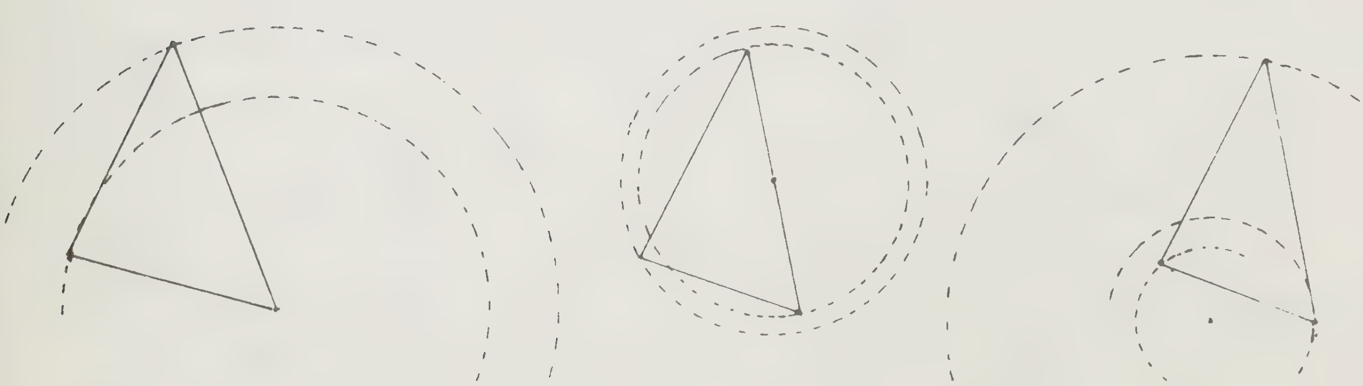


<u>slide in any direction</u>

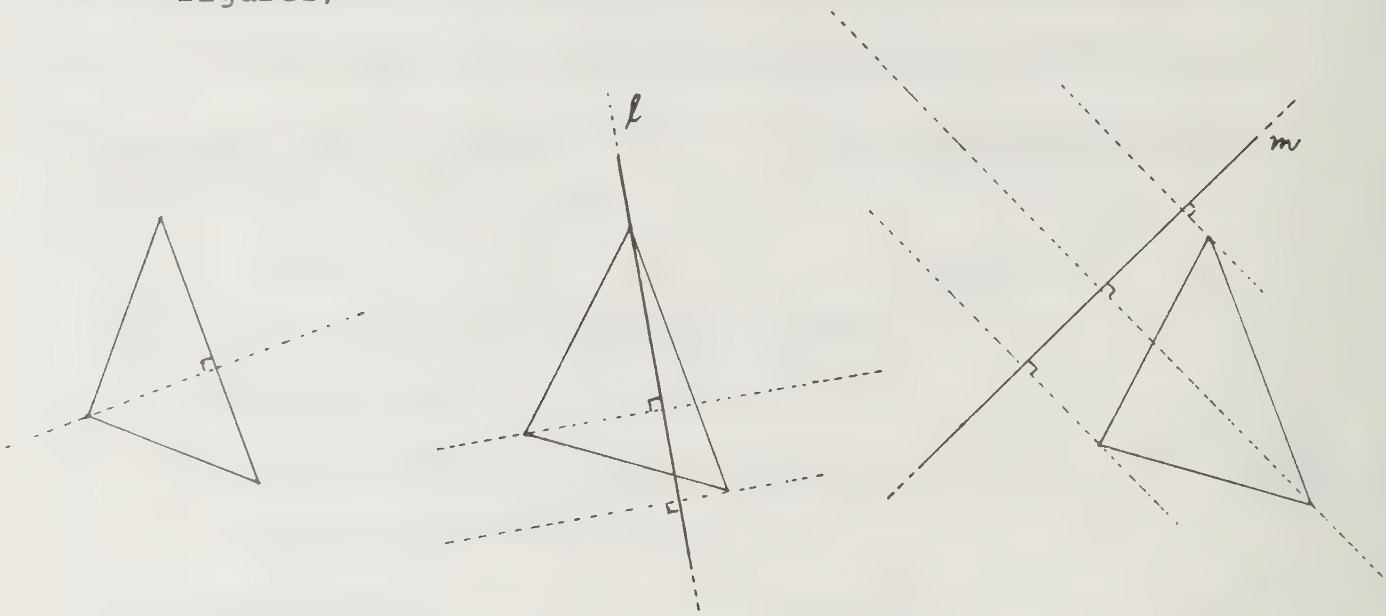


In each case, a before-after diagram can be drawn by tracing the figure, and then the geometric properties of the figures observed. To increase the accuracy of the diagrams:

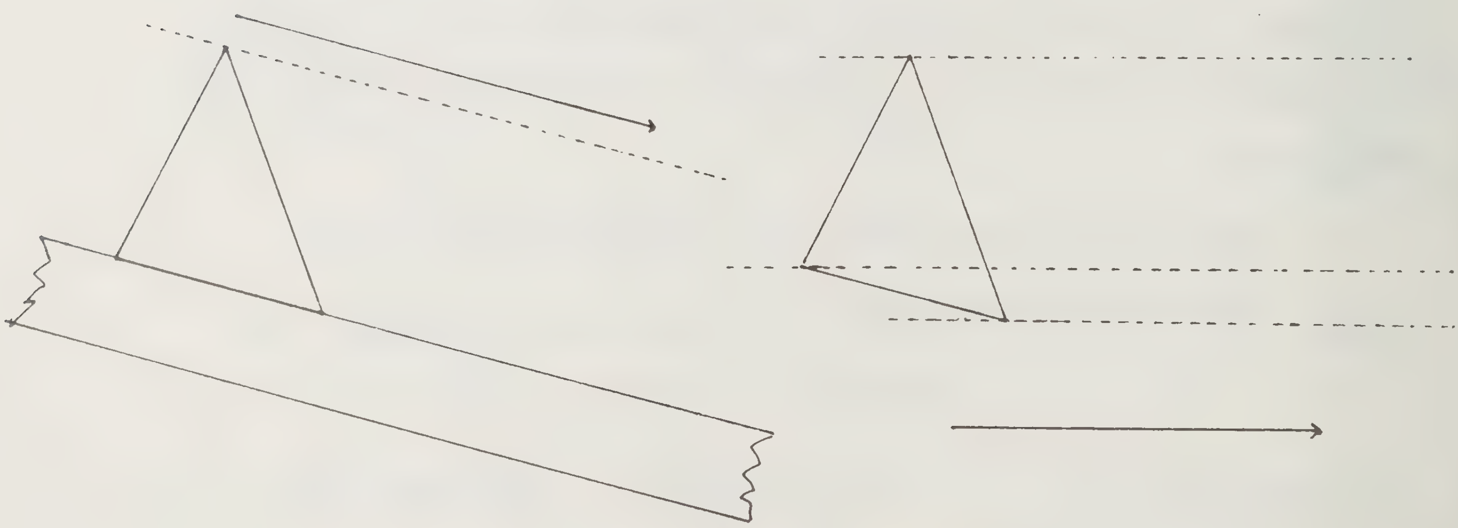
i) for turns, draw concentric circles through the vertices of the figure;



ii) for flips, draw perpendiculars to the flip-line with equal distance to the 'before' and 'after' figures;



iii) for slides, draw lines through the vertices parallel to the direction of the slide-arrow.

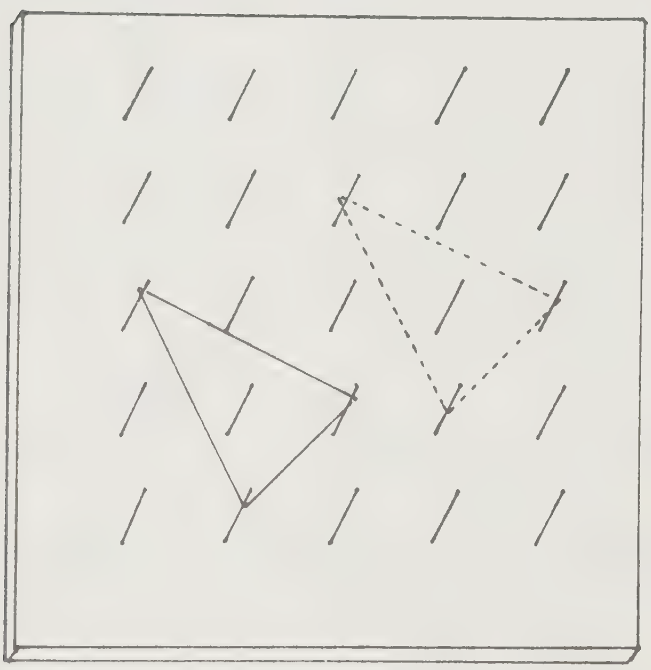


2. Activities with geoboards (square, equilateral, circular)

In activities with geoboards, a figure is formed by an elastic around a set of posts, and a particular motion is described. The student must visualize the motion as described, and construct the image.

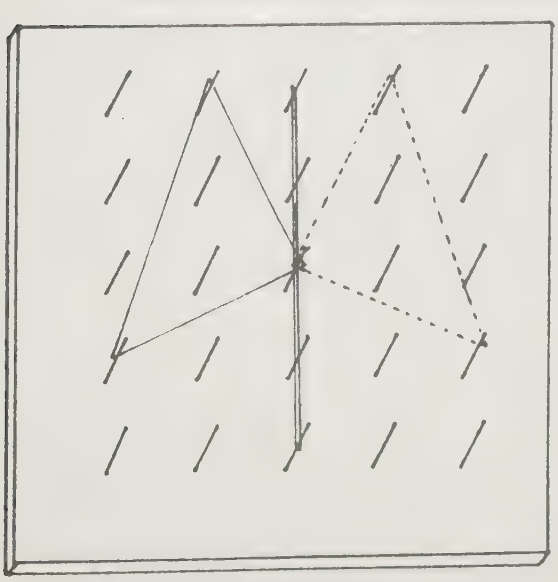
i) Slide images are easiest to obtain, and provide a good introduction to coordinate geometry.

slide (right 2, up 1)

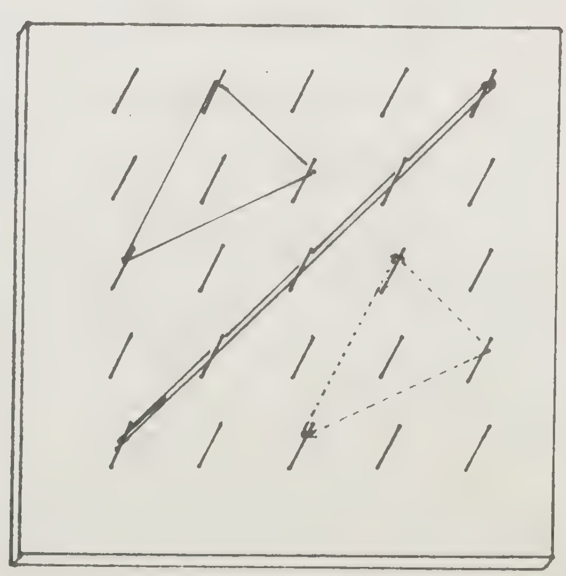


ii) Flip images are next in the degree of difficulty.

flip over the vertical



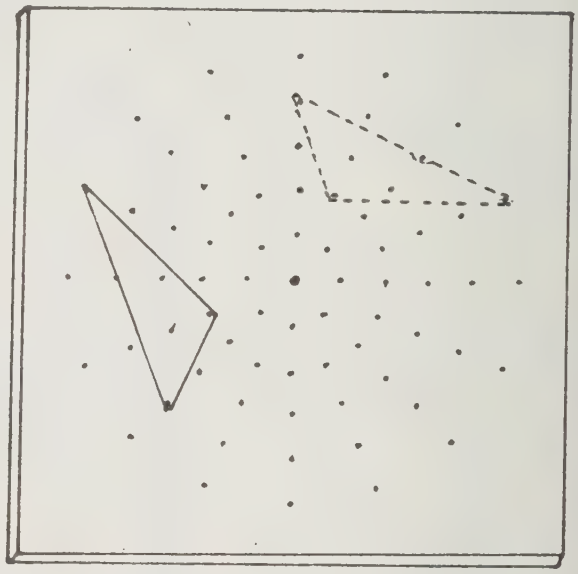
flip over the diagonal



iii) Turn images are more difficult than slides and flips for most students. Only $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ turns can be considered on a square geoboard. Use equilateral and circular geoboards for a wider range of turns.

Turn 90° clockwise

Turn 135° clockwise



Students should make diagrams on dot paper or grids to represent the geoboard results. At the simplest level, the scale on the geoboard and the dot paper is the same. At a later stage, the size of the figures may be reduced on the dot paper.

Many books dealing with geoboard activities are available from distributors of mathematical manipulatives, and many excellent articles have appeared in recent years dealing with activities and problems to be solved using geoboards.

3. <u>Activities using dot paper or grids</u>

   Dot paper or grids may be used to investigate the basic ideas of 1 and 2 above without first using tiles or a geo-board. These activities are more abstract than those with the physical materials, but still are very real to the students.

4. <u>Activities with paper folding, paper folding and cutouts, and paper folding and carbon paper</u>

   Activities with paper folding can produce:

   - flip images by means of one fold;
   - slide images by means of 2 parallel folds;
   - turn images by means of 2 non-parallel folds.

   These become largely recognition exercises; the motion can be verified by using tracing paper as described below.

   There are many excellent books, paperbacks, and articles in professional journals that deal with this topic.

5. <u>Activities with mirrors and transparent mirrors</u>

   Plane mirrors are used:

   - to visualize images;
   - to explore line-symmetry.

   Transparent mirrors are used:

   - to see visual reflection-images;
   - to draw these images;
   - to show that paper-folding images (or flip-images) are the same as the visual reflection-image;
   - to establish correspondences;
   - to establish properties;
   - to study line-symmetry.

Again, commercial material is available from a variety of sources.

6. <u>Activities with tracing paper</u>

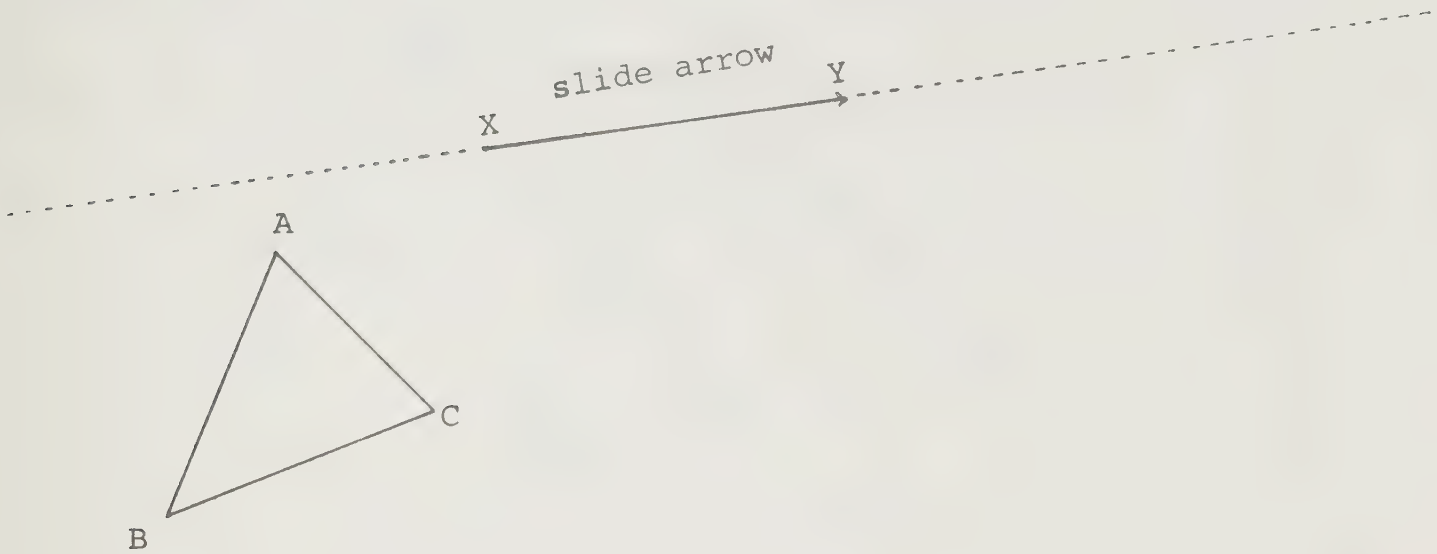
   Tracing paper may be used to:

   - construct slide-, turn-, and flip-images of figures. (The methods for this are discussed in a), b), and c) below.)
   - to show that two figures are related by one of these motions.

     (The methods for this are discussed in the notes for 8G 2e).)

a) Constructing a slide-image using tracing paper

Use tracing paper to construct the image of a figure under the slide with direction and distance indicated by the slide arrow.



Method

i) Extend the slide-arrow by a dashed line (as shown).

ii) Place tracing paper on the figure.

iii) Mark the positions of points A, B, C, and the arrow. Call the points A_1, B_1, C_1, X_1.

iv) Slide the traced-arrow along the dashed line until X_1 fits exactly on Y.

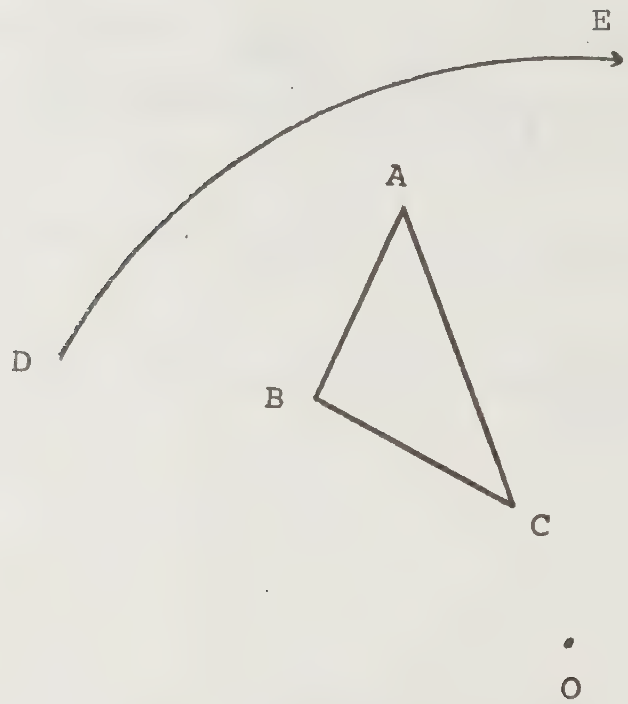
v) Mark the positions of A_1, B_1, and C_1 (pierce the page with a compass point).

vi) Draw triangle $A_1B_1C_1$, the required slide-image of $\triangle$ ABC.

The notation A_1 for the image of A, etc., should be introduced incidentally and read as 'A one'.

b) <u>Constructing a turn-image using tracing paper</u>

Use tracing paper to construct the image of a figure under the turn with centre O and the turn measure as indicated by the <u>turn arrow</u>.



<u>Method</u>

i) Place tracing paper on the figure.

ii) Mark the positions of points A, B, C, and D. Call them A_1, B_1, C_1, and D_1.

iii) Pin the tracing paper at O (using the point of a pencil or compass point).

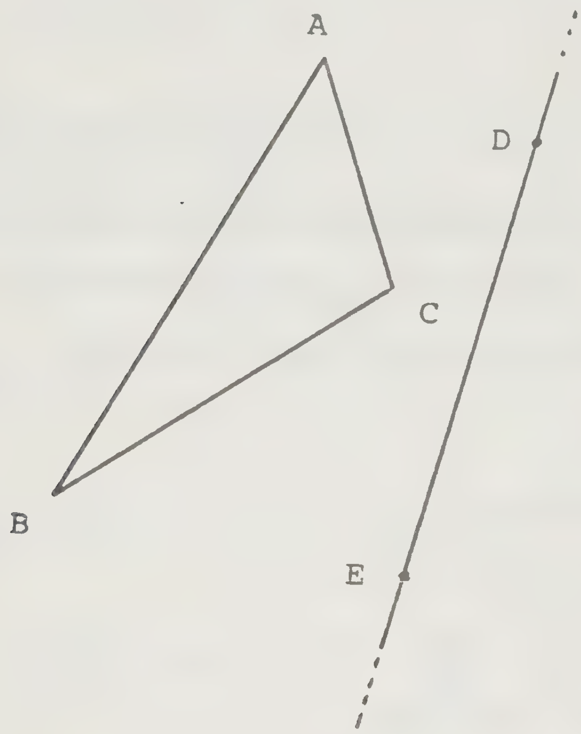
iv) Turn the tracing until D_1 is exactly over E.

v) Mark the positions of A_1, B_1, and C_1 (pierce the page with a compass point).

vi) Draw $\triangle A_1B_1C_1$, the required rotation image of $\triangle ABC$.

c) Constructing a flip-image using tracing paper

Use tracing paper to construct the flip-image of a figure under a flip over line.



Method

i) Place tracing paper on the figure.

ii) Mark the positions of points A, B, C, D, and E (D and E are any two points in the reflection line). Call them A_1, B_1, C_1, D_1, and E_1.

iii) Flip the tracing paper so that D_1 and E_1 fit exactly onto D and E.

iv) Mark the positions of A_1, B_1, and C_1 (pierce the page with a compass point).

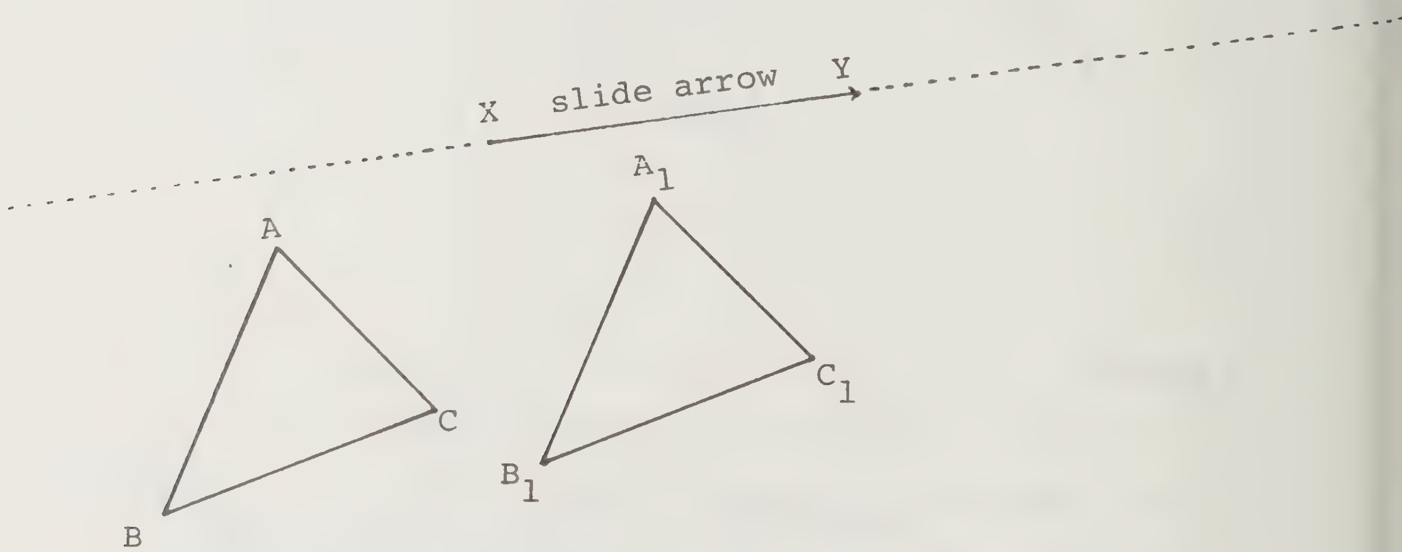
v) Draw $\triangle A_1B_1C_1$, the required flip image of $\triangle ABC$.

The above methods are simple to do, but they take much longer to describe. The focus should be on doing, not on a written or verbal 'blow by blow' description of the method. Students should be given enough practice in drawing slide-, turn-, and flip-images to ensure they are comfortable with these methods.

b) <u>Translation, rotation, and reflection related to the motions in a); the fundamental property of each</u>

Translation

Refer to example 6a) on page 11. In it, the slide-image of triangle ABC was located by moving the tracing paper according to the rule in step iv). The diagram below represents the before and after positions of Δ ABC in this motion.



Students should be asked to investigate how the corresponding points A_1 and A, B_1 and B, C_1 and C are related. To do this, have them draw arrows AA_1, BB_1, and CC_1. Compare their lengths and directions.

The students should find that arrows AA_1, BB_1, and CC_1 are equal in length, are parallel, and point in the same direction. The same situation is true for any pair of corresponding points in the two triangles.

When two figures are related by this property, we say that the second figure is a translation image of the first. The property is called the Fundamental Property of a Translation. It is summarized as follows.

Under a translation
the arrow from any point P to its image-point P_1 has the
same length and same direction
as the arrow from any other point Q to its image-point Q_1.

Teachers should be aware of the following comments.

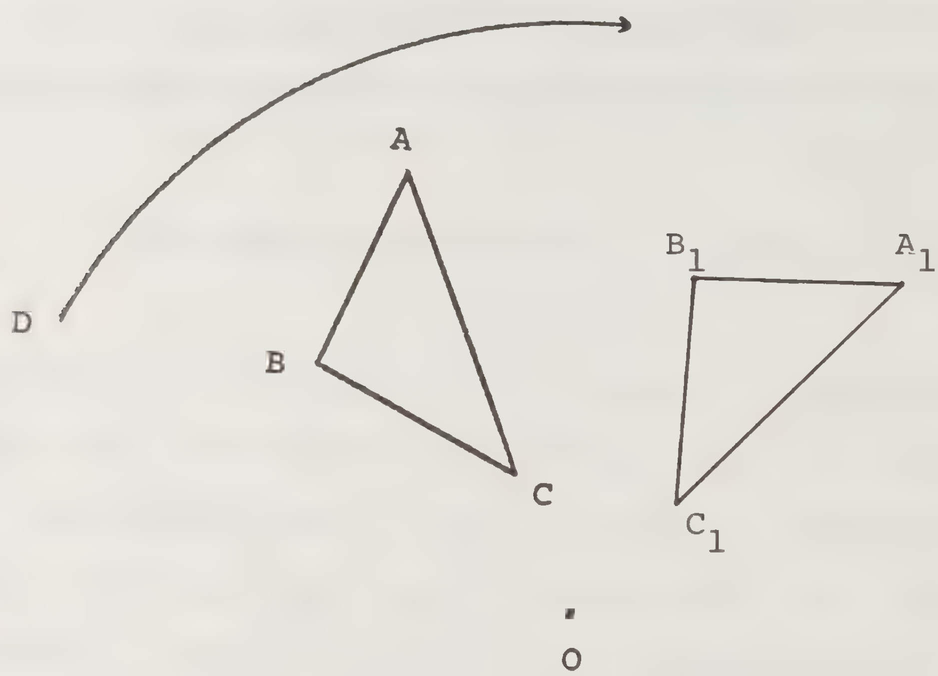
The word translation is used when the emphasis is on the mapping idea, that is on the correspondence from points in the object to points in the image. The word slide is used to describe the physical motion (of a tracing, a tile, etc.) by which the translation (correspondence) may be established. In describing the correspondence, we often say,

A maps onto A_1,
A translates onto A_1,
A_1 is the image of A, or
A_1 corresponds to A.

The notation $A \longrightarrow A_1$ means any of the above.

Rotation

Refer to example 6b) on page 12. In it, the turn-image of $\triangle$ ABC was located by moving the tracing paper according to the rule in step iv). The diagram below represents the 'before' and 'after' positions of $\triangle$ ABC in this motion.



Students should be asked to compare

i) the lengths of OA_1 and OA, OB_1 and OB, OC_1 and OC.

ii) the sizes of $\angle AOA_1$, $\angle BOB_1$, and $\angle COC_1$.

They should find the following:

i) $OA_1 = OA$, $OB_1 = OB$, $OC_1 = OC$

ii) $\angle AOA_1 = \angle BOB_1 = \angle COC_1$ = turn angle.

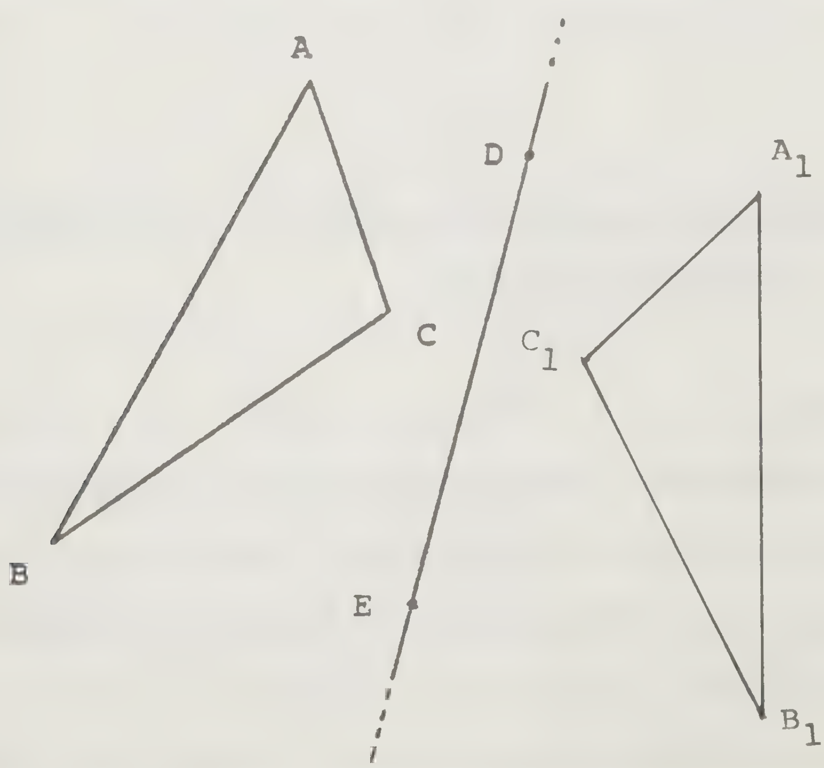
The same situation is true for any other pair of corresponding points in the object and image triangles.

When two figures are related in this way, we say the second figure is a rotation image of the first. The property is called the Fundamental Property of a Rotation. It is summarized as follows.

Under a rotation with centre O i) each point P in the object is the same distance from O as its image P_1 ii) angle POP_1 equals the angle of rotation for every position of P_1.

Reflection

Refer to example 6c) on page 13. In it, the flip-image of $\triangle$ ABC was located by flipping the tracing paper according to the rule in iii). The diagram below represents the before and after positions of $\triangle$ ABC in this motion.



Students should be asked to compare the positions of A and A_1, B and B_1, C and C_1 with respect to the flip-line.

They should find that line DE bisects segments AA_1, BB_1, and CC_1 at right angles. This situation is true for any other pair of corresponding points in the object and image triangles.

When two figures are related in this way, we say the second figure is a <u>reflection image</u> of the first figure. The property is called the <u>Fundamental Property of a Reflection</u>. It is summarized as follows.

> <u>Under a reflection in line m</u>,
> line m is the perpendicular bisector of the segment joining each point P to its image P_1 (m is the perpendicular bisector of PP_1).

In statement of the property above, it is assumed that P is not in the reflection-line. If it is, then P is its own image (it is invariant) -- this is important in later work, but may be overlooked at this stage if the students don't ask about it.

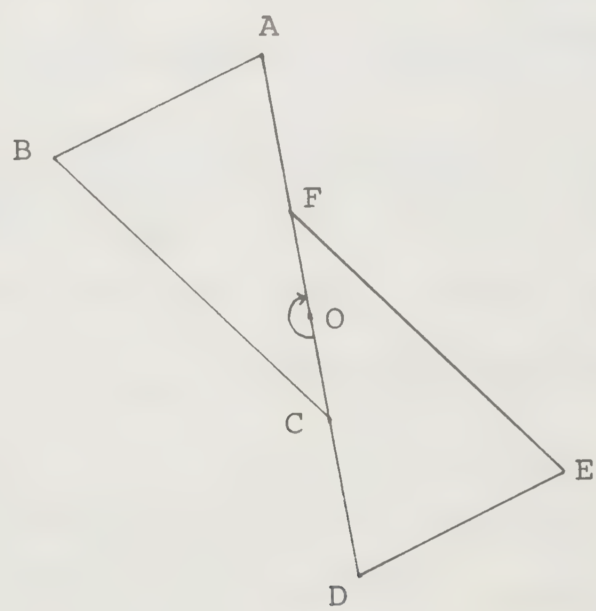
c) Constructing translation, rotation, and reflection images; congruence of corresponding points

Translation, rotation, and reflection images can be constructed by a variety of methods: using tracing paper, using ruler and compass, or using a transparent mirror. The tracing paper methods have been discussed in topic a) above; the other methods are discussed in the notes for 8G 2a). Each of these methods makes use of physical materials to construct the image of a figure according to the fundamental property of the particular transformation.

Students should practise construction of translation, rotation, and reflection images by one of these methods. The other methods can be developed in Grade 8. They should keep the fundamental property of the transformation in mind as they use the construction skills. They might ask, "why does this method produce an object-image figure with the desired property?"

The tracing paper techniques clearly establish the corresponding parts of the figures. Since the students have made frequent use of the tracing-paper test for congruence in the past, it is obvious that figures constructed with tracing paper are congruent. That is why it was used. Students should practise naming the corresponding parts in the language and symbols of mappings.

<u>Example</u>



Given that $\triangle ABC \longrightarrow \triangle DEF$ under a half-turn about O.

Make a tracing of $\triangle ABC$, labelling A,B,C, and O. Half-turn it about O to illustrate the correspondences.

Complete the following.

i) A ⟶

B ⟶

C ⟶

O ⟶

ii) AB ⟶ $\therefore$ AB $\cong$

OC ⟶ $\therefore$ OC $\cong$

OB ⟶ $\therefore$

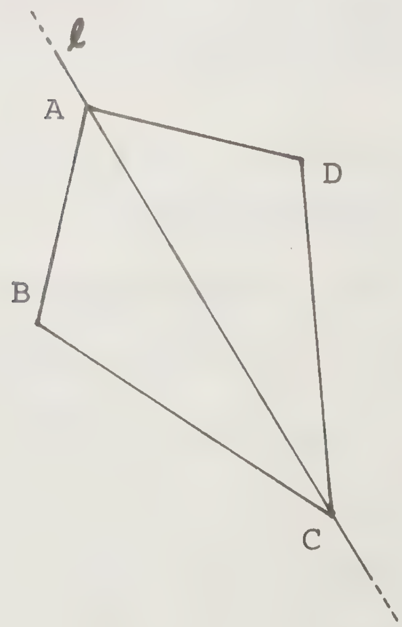
iii) $\angle$BAC ⟶ $\therefore$ $\angle$BAC $\cong$

$\angle$OBC ⟶ $\therefore$ $\angle$OBC $\cong$

d) Line-symmetry and rotational symmetry as mappings; congruence of corresponding parts

Line- and rotational symmetry were discussed in the notes for 7G 1c)d), 2a) and 3. At this stage, the relationships of corresponding parts under a symmetry can be described more explicitly with mapping notation.

Example



ℓ is a line of symmetry of quadrilateral ABCD (a kite). Make a tracing of quadrilateral ABCD and ℓ, labelling A, B, C, and D. Flip it over ℓ to illustrate the correspondences. Complete the following.

i) $A \longrightarrow A$

$B \longrightarrow D$

$C \longrightarrow$

$D \longrightarrow$

ii) Thus $AB \longrightarrow AD$ $\therefore AB \cong AD$

$BC \longrightarrow$ $\therefore BC \cong$

iii) $\angle BAC \longrightarrow$ $\therefore$ $\cong$

$\angle ABC \longrightarrow$ $\therefore$ $\cong$

$\angle BCA \longrightarrow$ $\therefore$ $\cong$

iv) $B \longrightarrow D$ and $D \longrightarrow B$

How is ℓ related to segment BD?

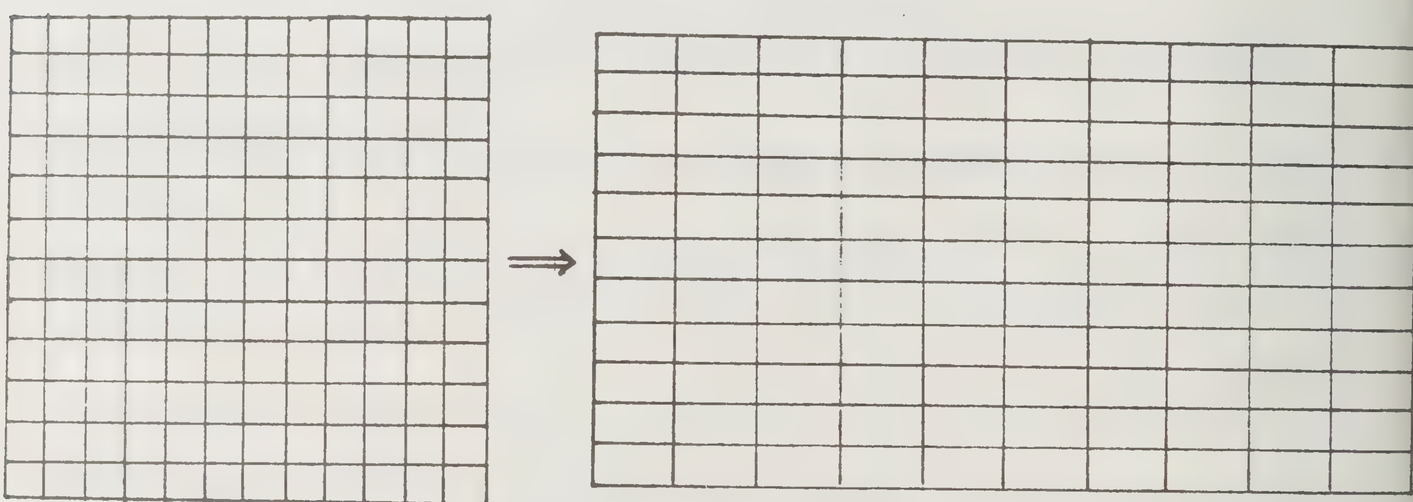
The following comments summarize statements made earlier in the notes. These situations could be re-examined, making use of mapping notation to describe correspondences and congruent parts.

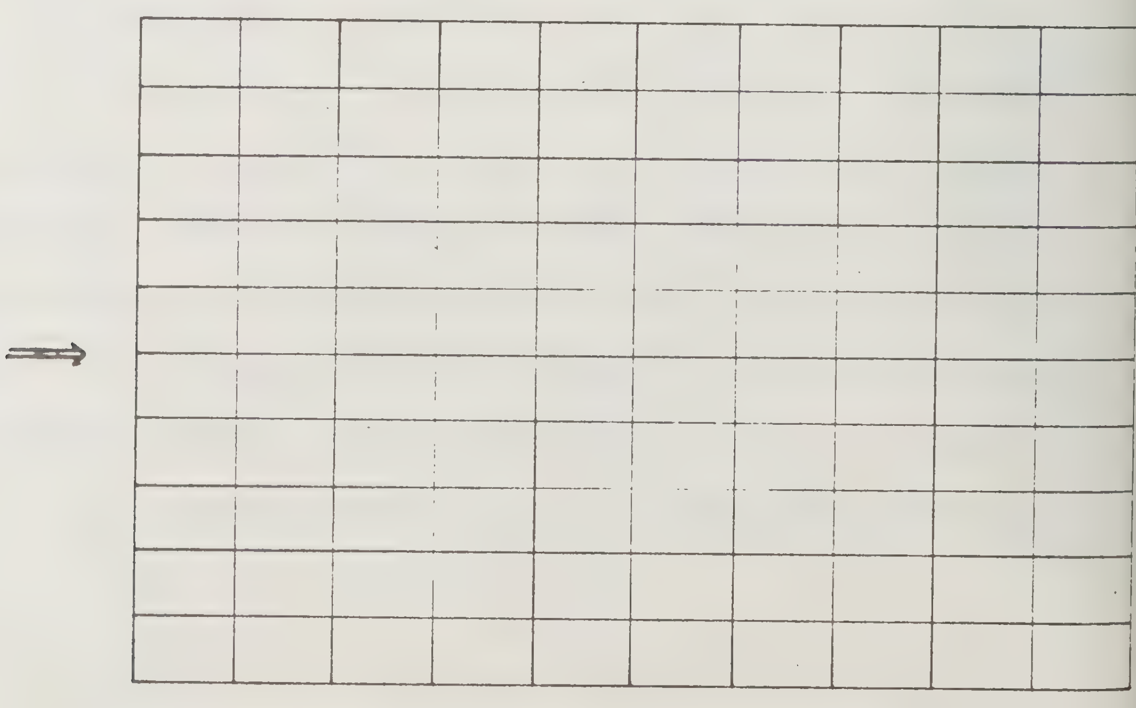
If a tracing of a figure can be fitted back onto itself in a new position, then the figure has symmetry - one symmetry for each new position. If the tracing has not been turned over, then the figure has rotational symmetry. If the tracing has been flipped, then the figure has line-symmetry. The line of symmetry (or lines of symmetry) and the centre of rotation can be located with tracing paper or with a transparent mirror. With line-symmetry, corresponding parts can be marked congruent.

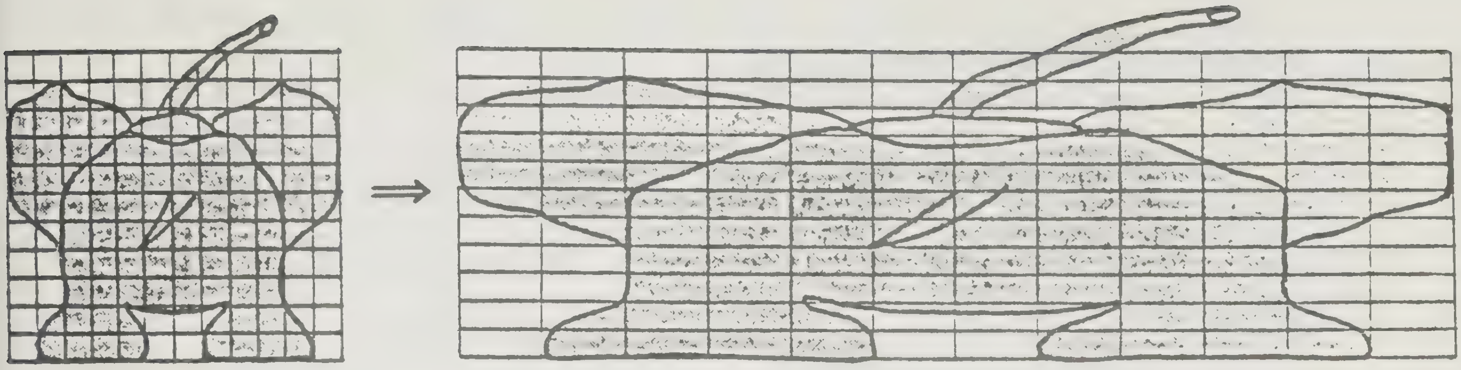
e) <u>Given a figure on a square grid, drawing its image on a distorted grid</u>

A deeper feeling for point mappings (the correspondence of points) can be engendered by starting with an object on a square grid and drawing its image on a distorted grid. Simply match the corresponding intersection points of object and image grids. Use a distorted grid composed of i) rectangles, ii) parallelograms, iii) other forms. The images are unpredictable, and fun to draw. The true concept of mapping is firmed up as a correspondence, divorced from physical motion.

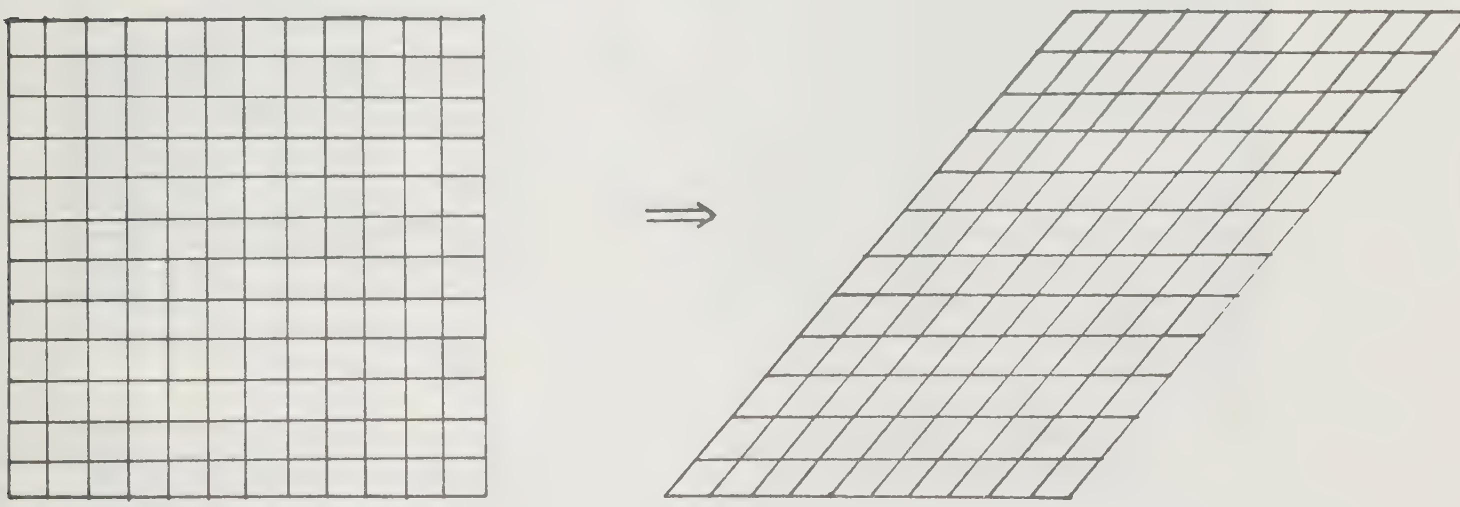
i) Square Grid ⟹ Rectangle Grid

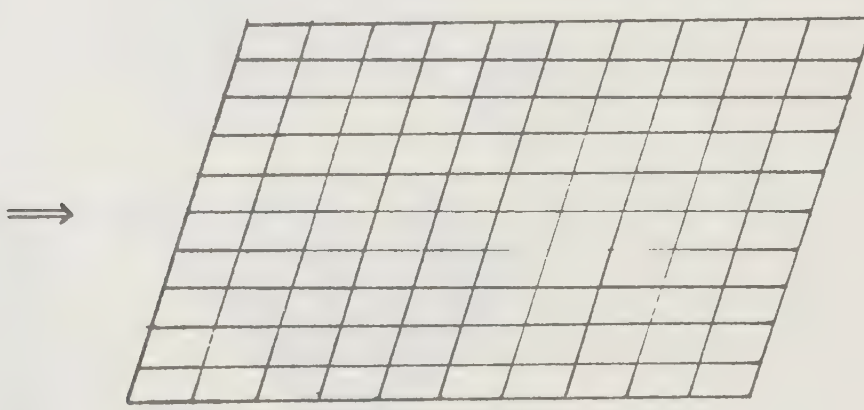


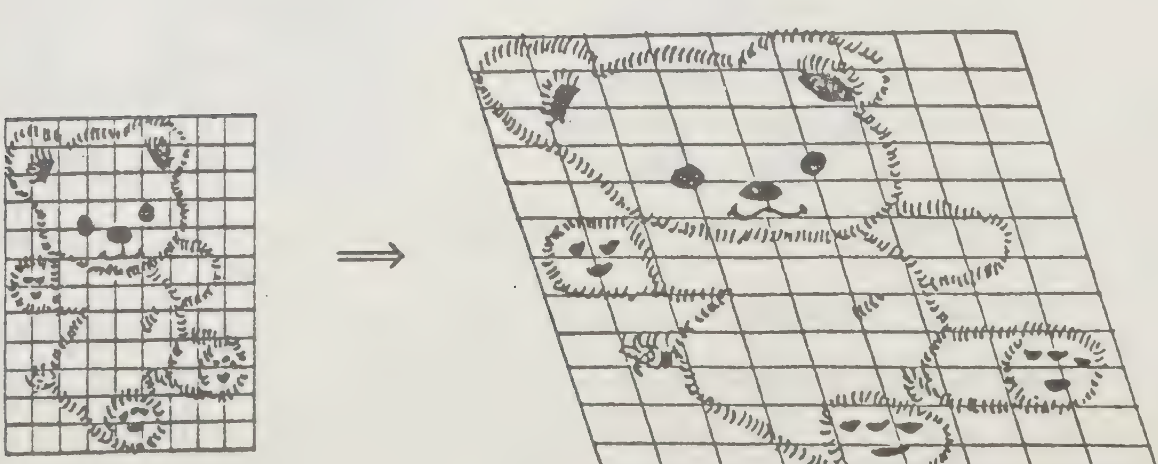




ii) Square Grid ⟹ Parallelogram Grid

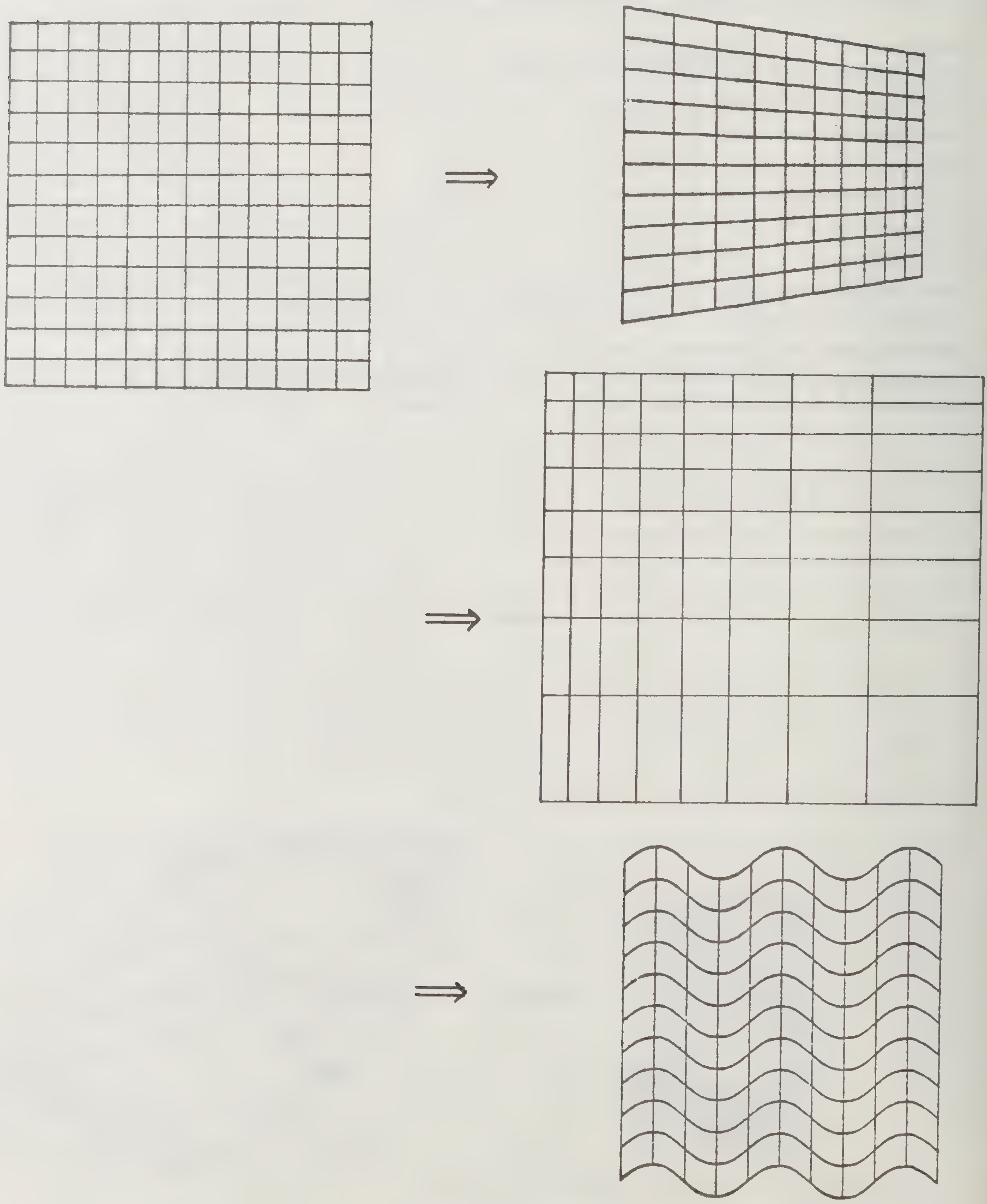




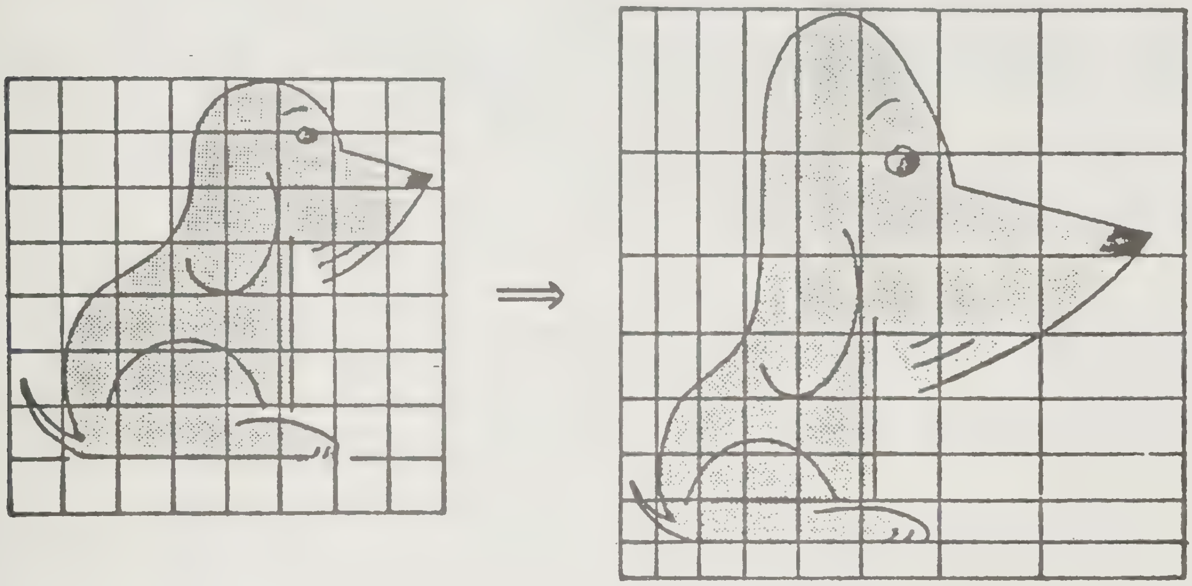


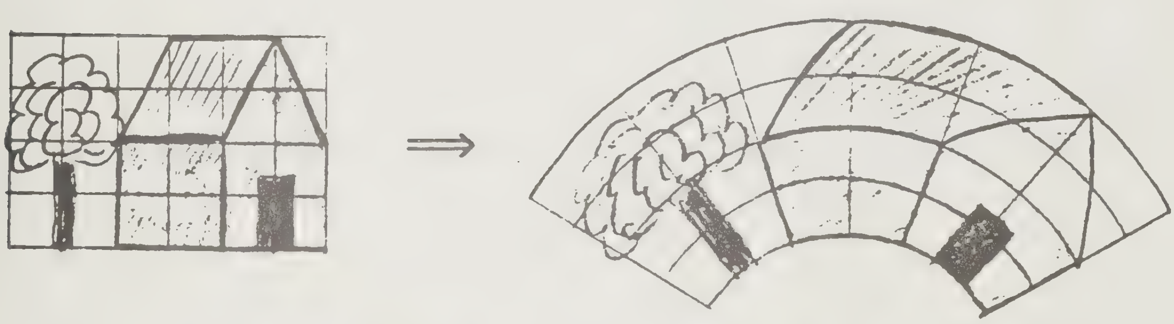
iii) Square Grids ⟹ Other Grids

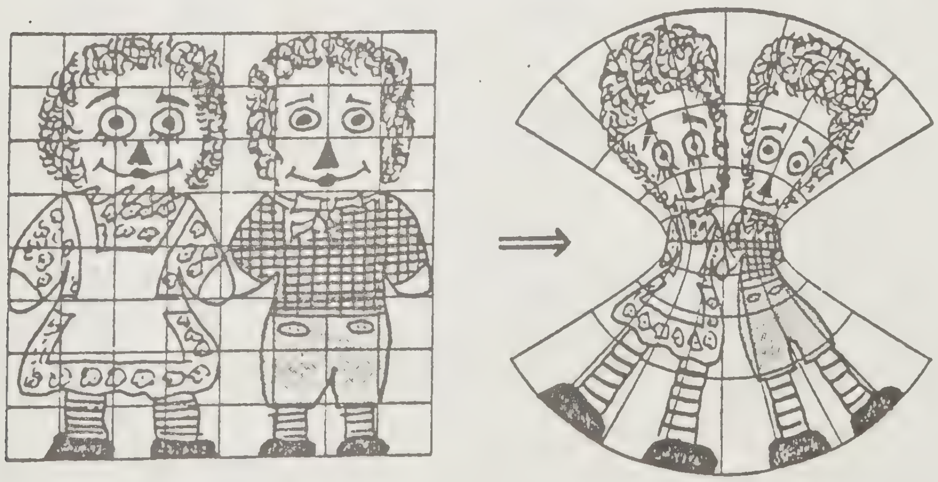
The lines on the second grid do not have to be the same distance apart, they do not have to be perpendicular or parallel, they do not even have to be straight. A few distorted grids can be prepared for the students to get them started; some students may want to create their own grids. Sample grids are shown below.



Students find graphing on distorted grids easy to do; yet there is significant mathematical value in these activities, initially in locating points on the grids and observing changes of position, orientation, size, and shape, then in identifying the characteristics of the original drawing that have changed and those that have remained unchanged.







The ideas, grids, and art work for the notes on distortions were submitted to the committee by Bob Alexander, Head of Mathematics, Eastern High School of Commerce, Toronto Board of Education.

Ministry of Education Ontario

GRADE 7 GEOMETRY

SECTION 5: ENLARGEMENTS AND REDUCTIONS

RELATED SECTIONS AND TOPICS

PAST	FY:	Pages 7 and 12; similar figures, maps, scale drawings
	Ed PJ Div:	Pages 74,75
PRESENT	Gr 7:	N 4; N 5b; N 8acde; A 1b; G 1abce; G 2; G 3; G 4; G 6
FUTURE	Gr 8:	N 1bdf; N 2abc; A 2; G 1a; G 2ab; G 3a; G 4; G 5
	Gr 9 Gen:	N 1; N 3a; N 5bc; N 6abde; A 2d; A 4a; G 2; G 3; G 4
	Gr 9 Adv:	N 5bf; N 3ad; A 4; G 2; G 3a; G 4
	Gr 10 Gen:	N 1a; N 2a; N 4a; N 7a; A 1b; G 1b; G 2
	Gr 10 Adv:	N 1c; N 2a; G 1a-e; G 2ab; G 3b; G 4; G 6abd

a) Enlarging and reducing geometric figures using geoboards, dot paper, tiles, or grids; shape, size, other properties, similar figures

The following notes suggest a strategy for developing this topic using a concrete stage working with geoboards and/or tiles, then a representation stage using dot paper or grids.

The Concrete Stage (using geoboards)

Activity 1 (Provide each student, or group of students, with a geoboard -- preferably 10 x 10.)

i) Make a small triangle on your geoboard. Call this the object. (See figure 1.)
This step may be demonstrated on an overhead projector, or with a classroom demonstration model. Each student should make a different triangle.

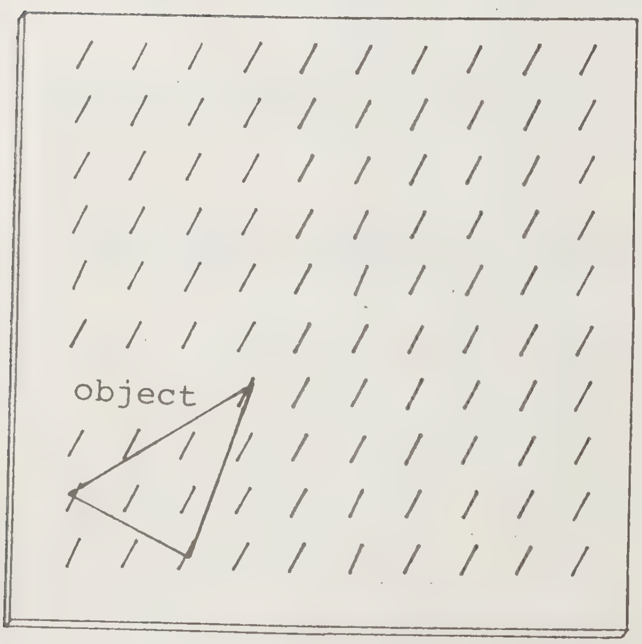


figure 1

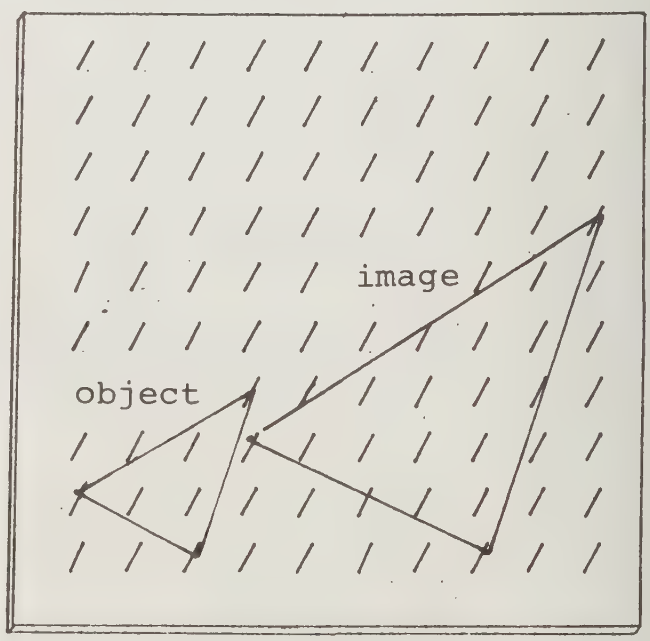


figure 2

ii) Now make a larger triangle that looks the same (has the same shape) as the first triangle. Call this the image. (See figure 2.)

iii) Compare:

- the <u>lengths</u> of <u>corresponding sides</u> of the two triangles. (This suggests the idea of scale ratio or scale factor. Each side in the image is twice as long as the corresponding side in the object. The scale ratio is 1:2 from object to image. The scale factor is 2.)
- <u>sizes</u> of <u>corresponding angles</u> of the two triangles. (The angles in each pair should look the same. Use a protractor for approximate readings.)
- areas of the two triangles. (Do this by counting squares and parts of squares; by formula if known at this time; by splitting the larger triangle into triangles congruent to the object; (this technique is not possible for all figures), or by Pick's Formula (see page 8).

<u>The Representation Stage</u>

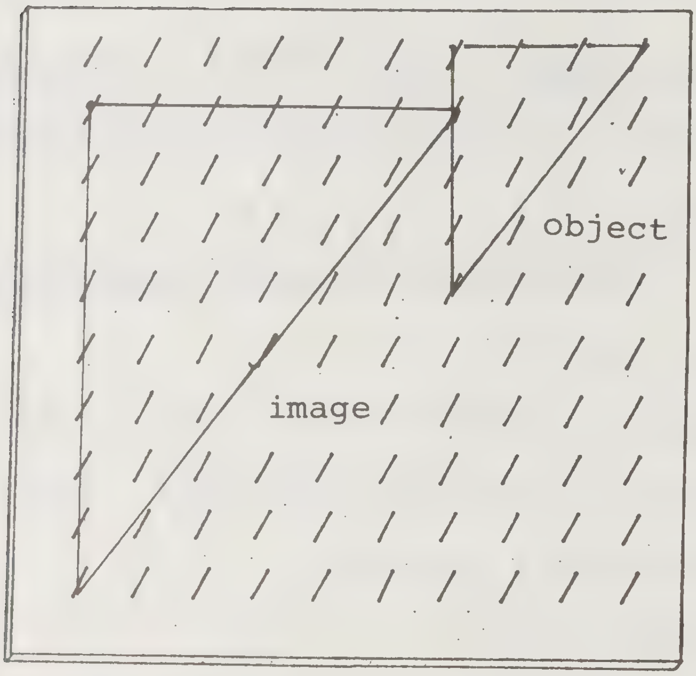
(Provide each student with dot paper, with pattern the same size as on the geoboard, as illustrated below.)

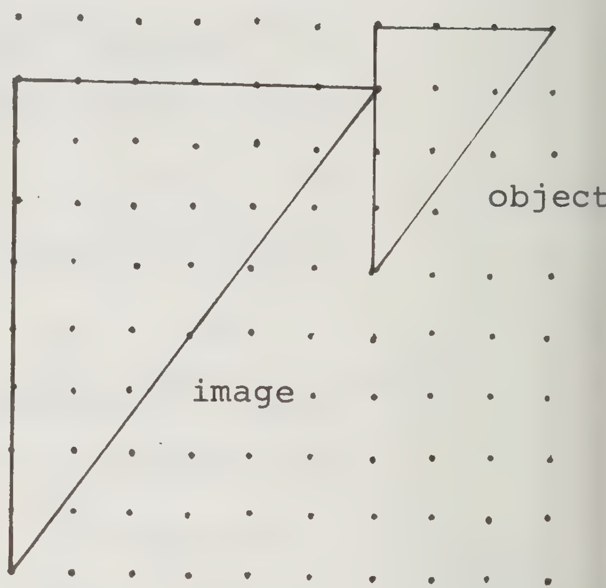
iv) Make a drawing on the dot paper to represent the pattern on the geoboard.

v) Using this diagram, compare:

- <u>lengths</u> of <u>corresponding sides</u> (See notes 7G 2a).)
- <u>sizes</u> of <u>corresponding angles</u> (See notes 7G 2a).)

- areas of the object and image triangles (as in step iii), or by using tracing paper to show how many times the object fits into the image -- slides, turns).





Object and Image on a Geoboard

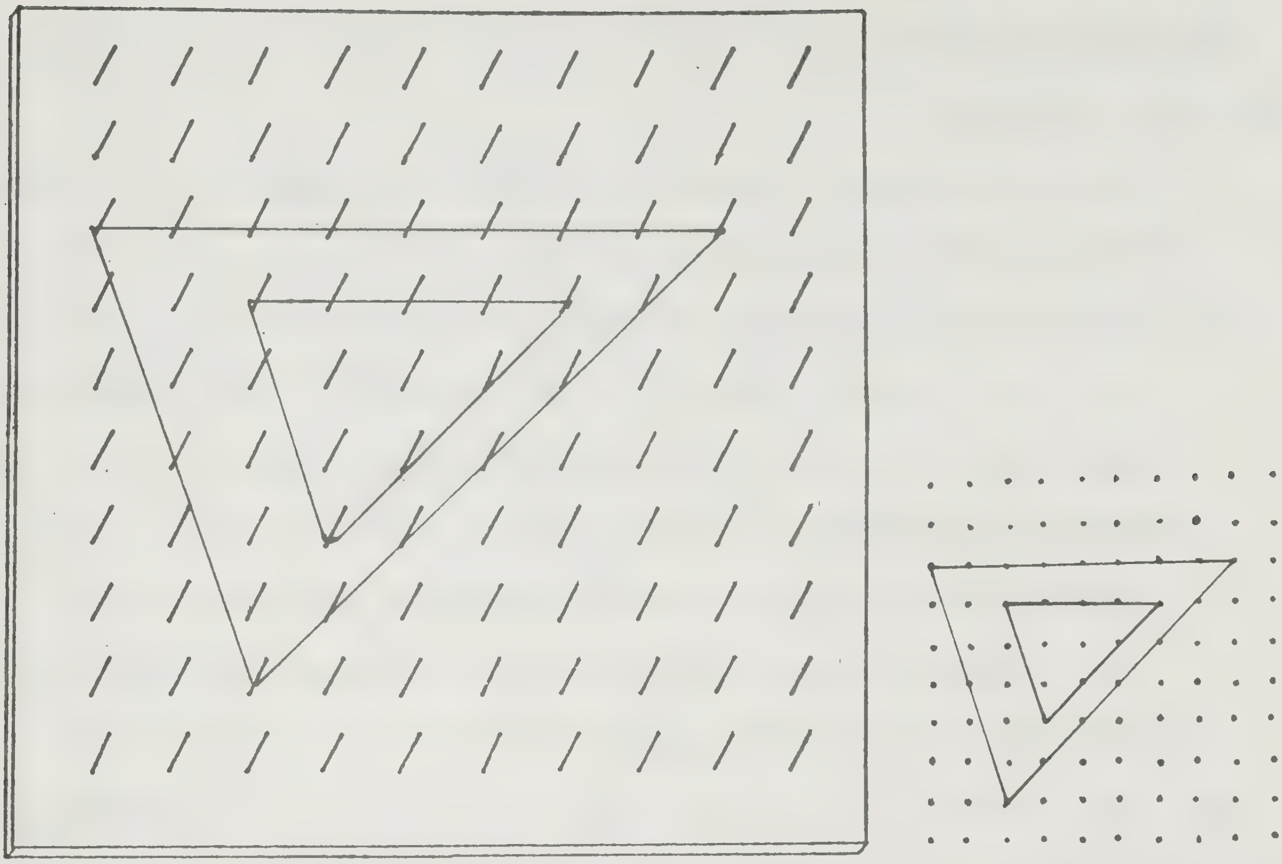
Object and Image Represented on Dot Paper

(Provide each student with dot paper in which the pattern is $\frac{1}{2}$ the size of the pattern on the geoboard.)

vi) Make a drawing on this dot paper to represent the pattern on the geoboard (as illustrated below).

vii) Using this diagram, compare:

- length of each side in the drawing with the length of the corresponding side on the geoboard
- sizes of the angles in the drawing with the corresponding angles on the geoboard
- areas of the triangles in the drawing with the areas of the corresponding triangles on the geoboard



In step vi), each student made a scale drawing of the geoboard pattern. The scale ratio is 1:2 from image to object; the scale factor is $\frac{1}{2}$ from object to image. The drawing establishes a mapping (correspondence) between the diagram and the geoboard figure. Many properties of the geoboard figure are preserved in the diagram, but not length or area.

In the above investigation, the students have either been using many important mathematical ideas directly, or have been exposed to them at an intuitive level. Some of these are:

- mappings, correspondence of points;
- counting posts to double or triple lengths of sides; and comparing lengths;
- comparing the sizes of angles;
- intuitive ideas of slope; 'over one up two' becomes 'over two up four' or 'over three up six';
- intuitive ideas of coordinates;
- ratio, shape, size, parallel segments, scale factor, etc.

The method of Activity 1 can be repeated for a number of geometric figures:

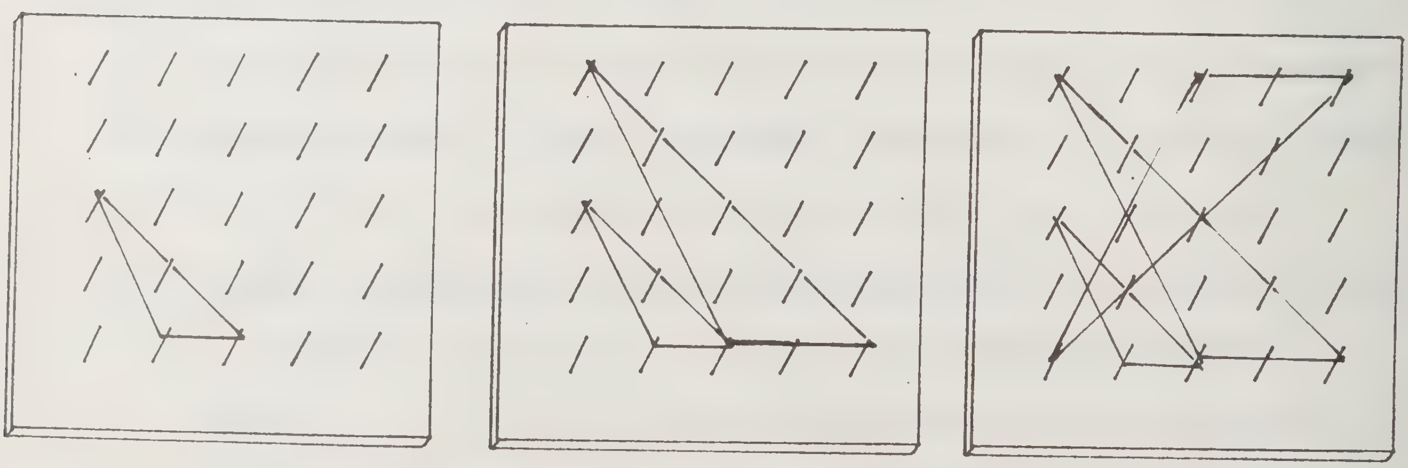
- other triangles, squares, rectangles, rhombi, parallelograms, and regular hexagon (on an equilateral geoboard).
  When these figures are enlarged by scale factors of 2,3,4,...,
  the image can be divided into parts that are congruent to the object. This is not true for most other figures, such as a quadrilateral.
- other closed figures of the students' creation
  (For these figures, refer to Pick's Formula on page 8 in order to compare areas.)

There are a number of ways by which Activity 1 can be expanded. Some of these are suggested by Activities 2, 3, and 4 below.

Activity 2 (Provide each student with a 5 x 5 geoboard.)

i) Repeat steps i) and ii) of Activity 1. A typical result is shown in the first two diagrams below.

ii) Make another triangle congruent to the image. One possible result is shown in the third figure below. (It is a reflection-image of the other image.)



etc

iii) Repeat step ii) to produce a different image.

iv) How many image triangles can be made congruent to the original image triangle? Draw them on dot paper. (The answer

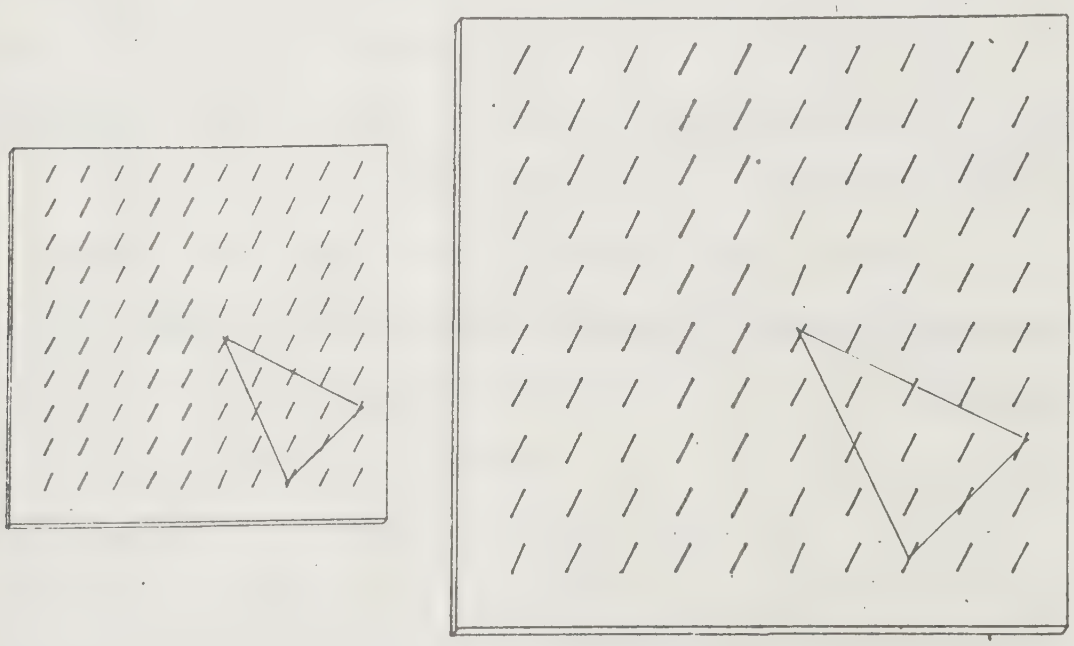
will depend on the figures in step i) and the size of the geoboard. The new figures will be reflection, rotation, or slide-images (or a combination).

Note: Activity 2 could be done directly on dot paper. However, when using the geoboard, the student can explore, make mistakes, and modify the figures without erasing.

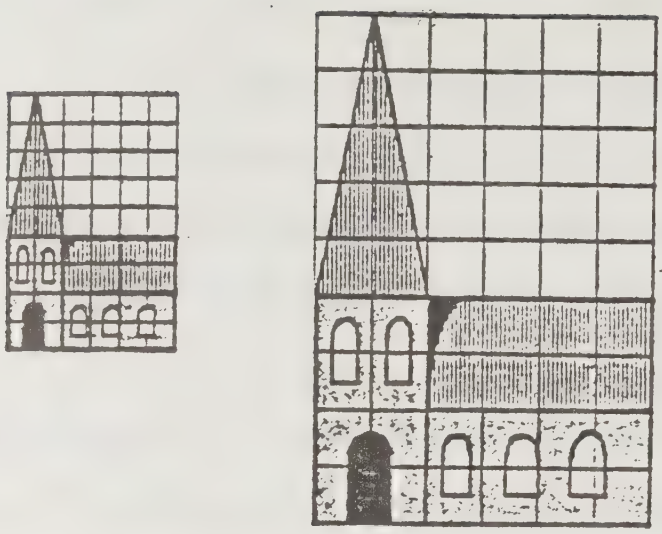
Activity 3 (Provide the students with two geoboards, in which one is constructed to a different scale than the other, as illustrated below.)

i) Make a figure on one geoboard, reproduce it on the other geoboard.

ii) Compare lengths of corresponding sides, determine the scale ratio, compare the sizes of corresponding angles, and corresponding areas.



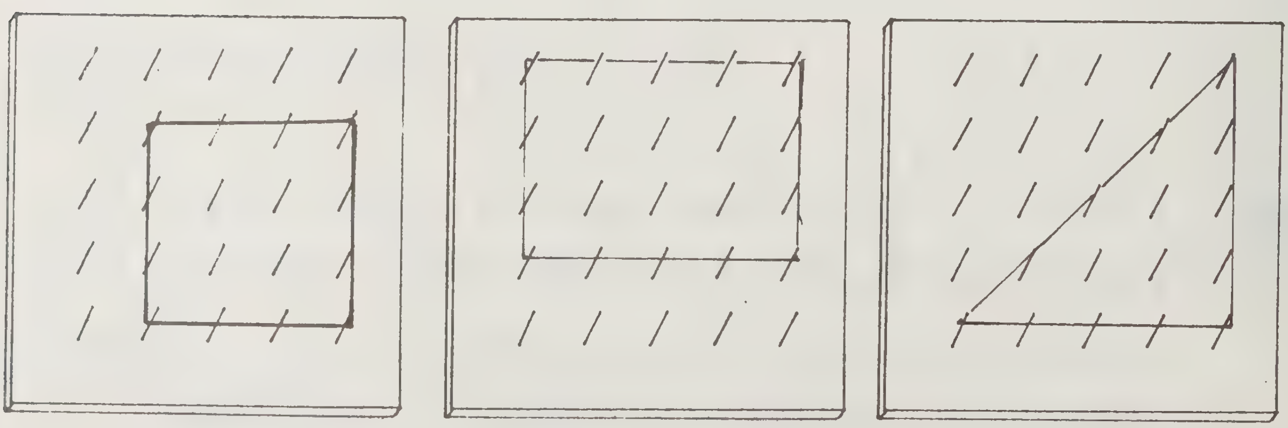
Note: Activity 3 could be done directly on two sizes of grid paper. The object could be any figure -- a house, dog, cartoon character, etc. The image will be an enlarged (or reduced) replica of the object -- same shape, new size.



The next activity introduces Pick's Formula for the area of a figure made on a geoboard (or on dot paper, or on a grid). The formula can be found experimentally, as suggested below. Once the formula is known it can be applied to the enlargement-reduction activities as a means of comparing areas.

Activity 4 -- Pick's Formula (Provide each student with a geoboard and dot paper.)

i) On a geoboard, make simple figures such as a square, rectangle, and right-angled isosceles triangle (as illustrated below).



ii) For each figure:

- find its area (count squares or calculate);
- count the total number of posts in the sides of the figure;
- count the total number of posts in the interior of the figure;
- record the information in the chart.

Figure	Area A	Number of Posts in the Sides (P)	Number of Posts in the Interior (I)
Square			
Rectangle			
Triangle			

iii) Try to find a rule to convert P and I into A; for example, with the square to convert 12 and 4 into 9, with the rectangle to convert 14 and 6 into 12, with the triangle to convert 12 and 3 into 8.

<u>Answer</u> $\frac{12}{2} + 9 - 1$, $\frac{14}{2} + 6 - 1$, $\frac{12}{2} + 3 - 1$. In general $\frac{P}{2} + I - 1$; this is <u>Pick's Formula</u>.

iv) Make other squares, rectangles, and triangles on your geoboard. Test Pick's Formula for each of these.

v) Does Pick's Formula work for other figures? Draw other polygons for which you can find the area by counting squares or subdividing the polygon. Test Pick's Formula on these polygons and record your results.

Example:

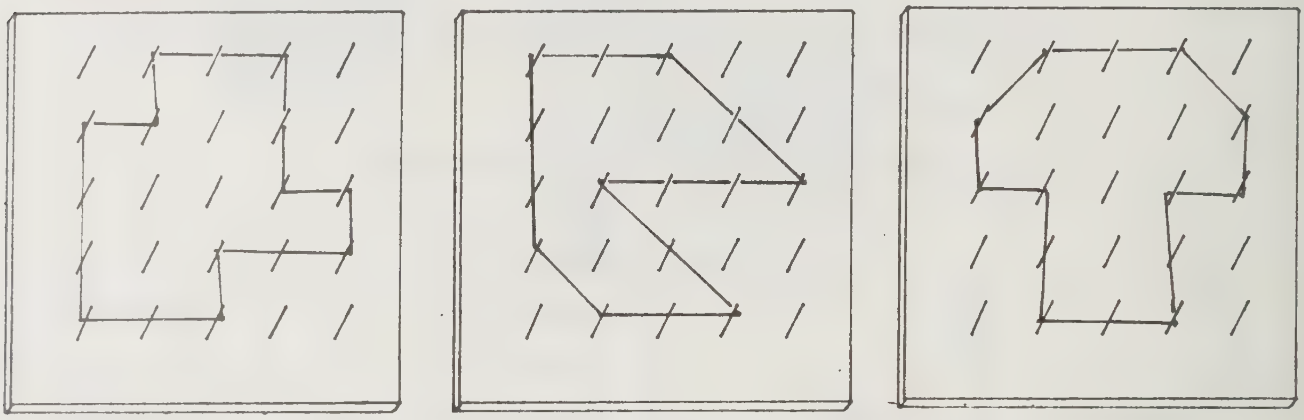
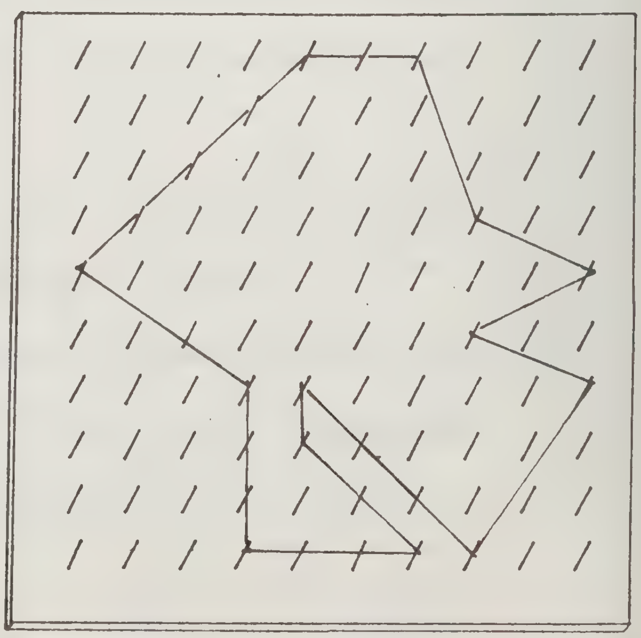
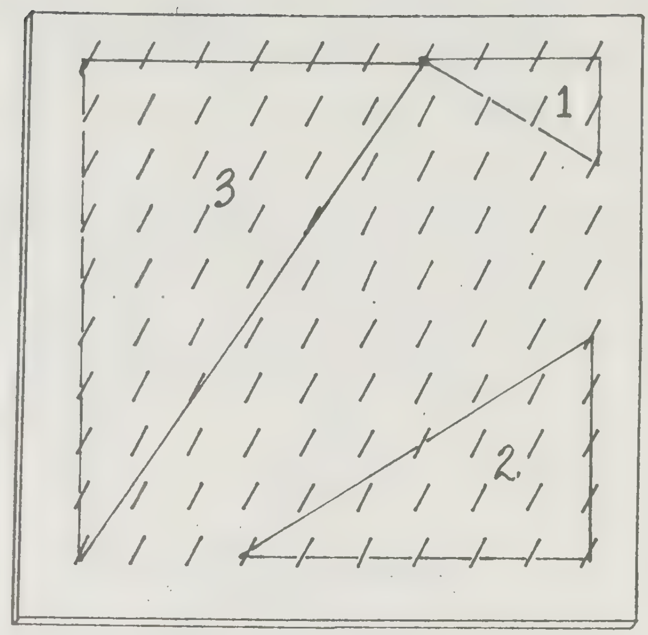


Figure	P	I	$\frac{P}{2} + I - 1$	A — by any other method
Polygon 1				
Polygon 2				
Polygon 3				
.				
.				
.				

vi) Now use a large geoboard and make complicated closed figures, such as the one shown on the right. Calculate the area of each figure.



vii) Use a large geoboard and make a number of figures with the same shape but different sizes. Calculate their areas. Compare the lengths of their sides. Find the scale ratio for each pair of figures; relate this to the ratio of their areas.

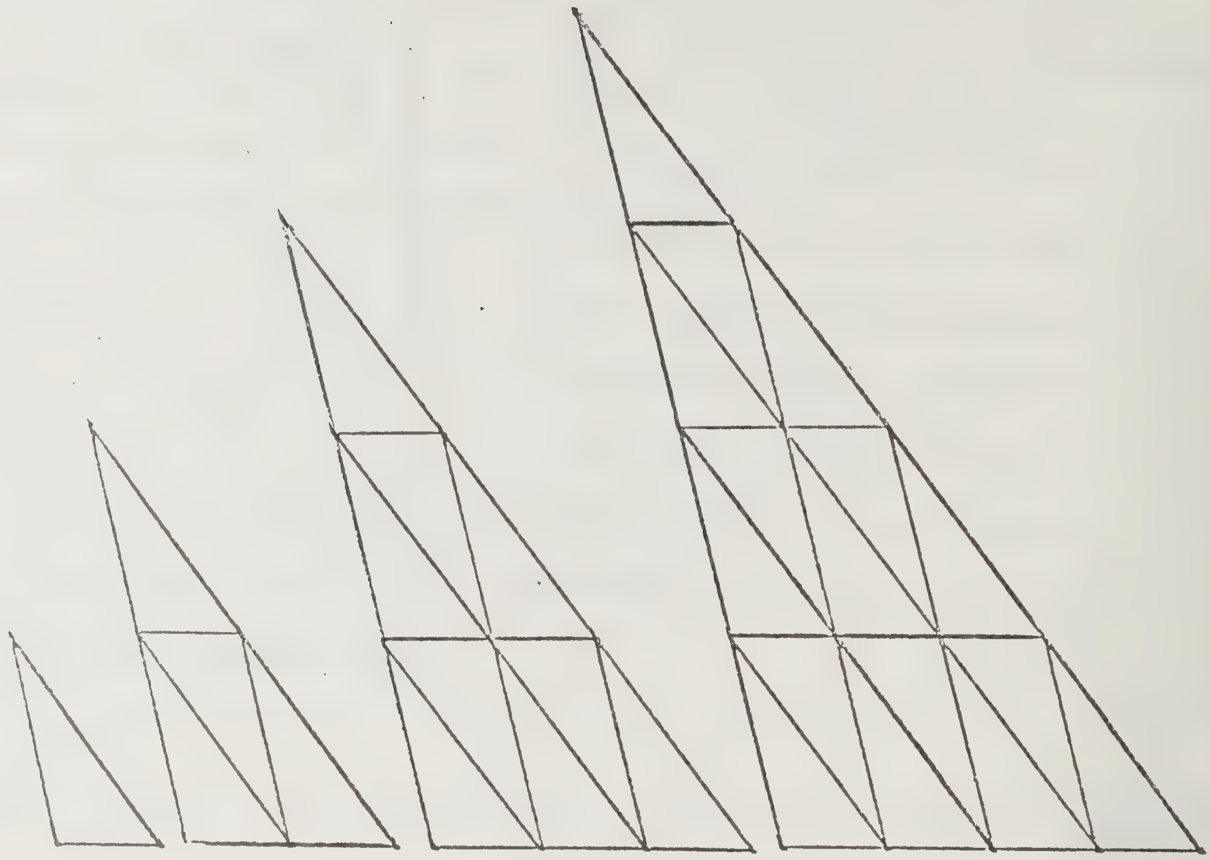


Tiles

Tiles may also be used in planning activities to illustrate figures with the same shape but different sizes, and to discover ratio properties relating corresponding sides, angles, and areas of these figures.

Activity 6 (Provide each student (of group of students) with sets of congruent tiles -- triangles, squares, rectangles, rhombi, and parallelograms.)

i) Use triangular tiles. Fit tiles edge to edge to form larger triangles that have the same shape (look the same) as the basic tile. (This is illustrated below.)



ii) Examine the triangles you have just made.

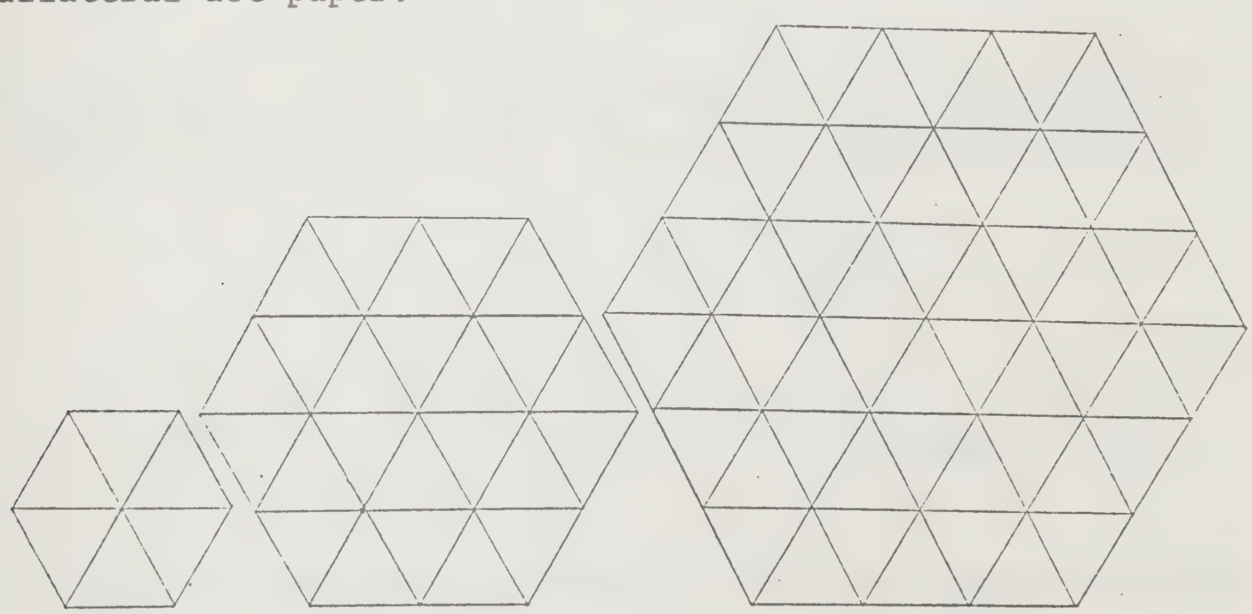
- Compare the lengths of corresponding edges. In what ratio are they? (1:2:3:4) Explain how you know this.
- Compare the sizes of corresponding angles of the triangles. How do you know they are congruent?
- Compare their areas (count tiles). In what ratio are they? ($1:4:9:25 = 1:2^2:3^2:4^2$) What would the area be of the next triangle? How does the area relate to the number of tiles along each side? What would the area be if there were k tiles along the base?

(When constructing the pattern from which the tiles are to be cut, mark the vertices to correspond with points on dot paper or a grid. In this way, you will have dot paper on which the tiling patterns can be easily reproduced. Provide students with this dot or grid paper.)

iii) Make diagrams on dot paper (or grid paper) to represent the triangles of step i).

This activity should be repeated for some of the other tiles mentioned above. Then it should be tried for a different tile, such as a scalene quadrilateral, to show that the patterns don't hold in general.

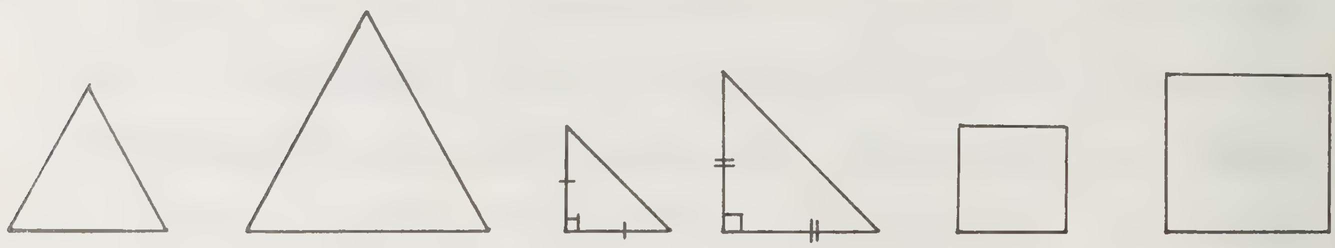
Activity 7 Plan an activity based on the following ideas. Equilateral tiles may be used to build a regular hexagon (of area 6) and in turn to build larger hexagons that 'look alike', as illustrated. The properties may be investigated as before. Diagrams for this case will need to be made on equilateral dot paper.



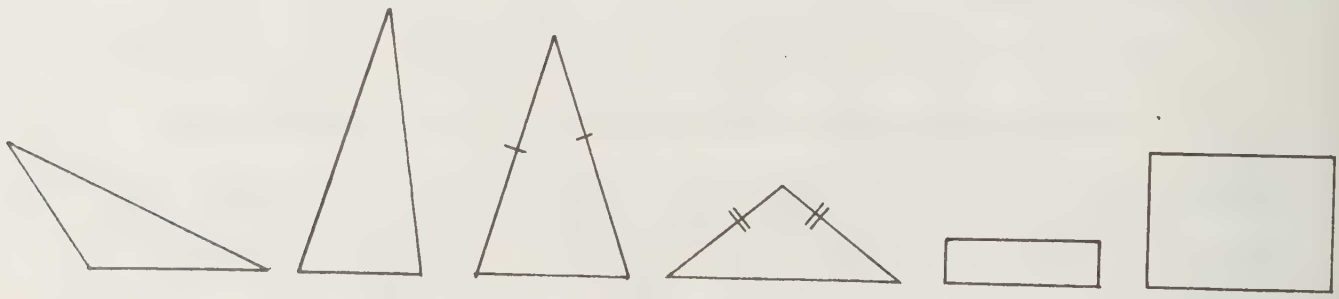
Shape, Size, Similar Figures

The above activities are designed to give the students experiences using concrete materials to build figures that have the same shape, but different sizes.

The word 'shape' is used informally in this topic to describe figures that 'look alike'; i.e. 'have the same shape'. In the formal sense, shape is an attribute (or characteristic) of a figure. Although it can be defined in terms of dilatation, this formalism is not desirable nor expected at this grade level. Its meaning should be established by using the word shape in correct contexts. For example, all equilateral triangles have the same shape, as have all right-angled isosceles triangles and all squares.



Different scalene triangles, isosceles triangles, and quadrilaterals other than the square may have different shapes.



The word shape should not be used as a synonym for figure, since it is an attribute of the figure, not the figure itself. The meaning of shape is refined further in the optional topics:

- 7G 4e), which deals with distortions, where the shape of the image is different than the shape of the object;
- 8G 2d), which deals with distorted images in curved and irregular mirrors;

and by discussion of real world examples of figures that have:

- the same shape -- scale drawings, photographic enlarge-

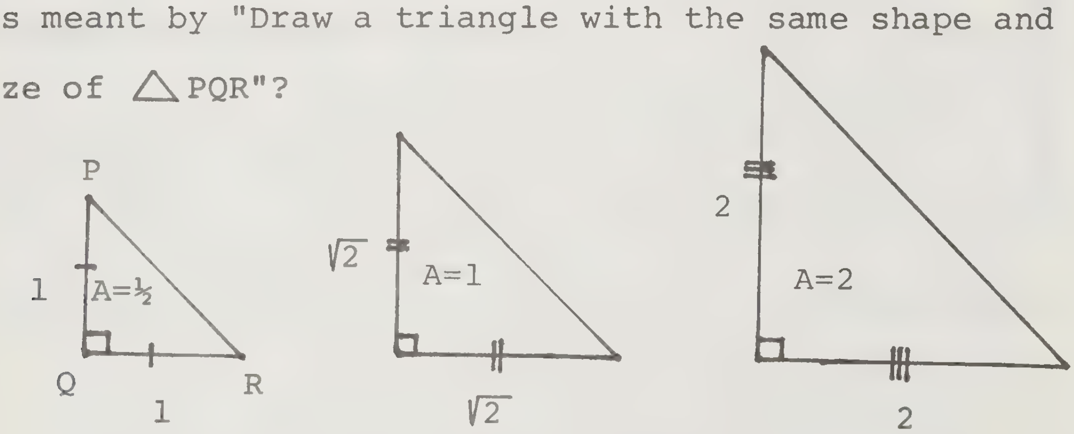
ments and reductions, micro fiche, patterns, etc.;

- different shapes -- shadows, overhead projector images, shadows created by lamp shades, perspective views of a circular ring, 'before and after' pictures in health club advertisements, and so on.

Size

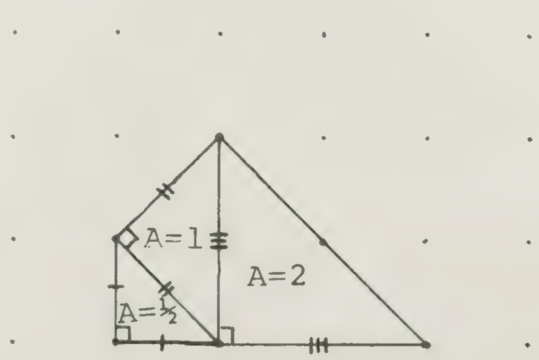
This word is difficult to define, since it has different meanings in different contexts. When referring to the size of a figure, we generally mean its area. In the tiling and geoboard figures illustrated earlier in this section, the image has the same shape as the object but different size. Different size in this context means different area.

What is meant by "Draw a triangle with the same shape and twice the size of $\triangle PQR$"?



In this context, size refers to the area of the triangle, not to the lengths of its sides. The first triangle has area $\frac{1}{2}$. The triangle with area of 1 is twice the size of the first triangle; the length of its equal sides is $\sqrt{2}$.

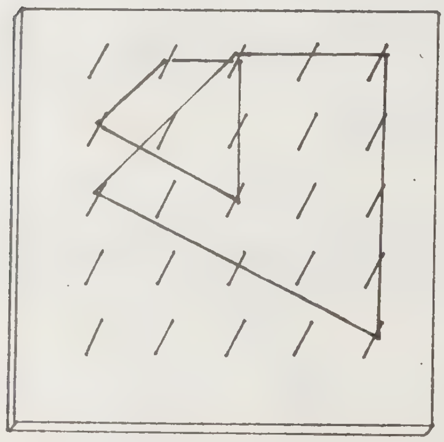
The above triangles may be constructed on a geoboard or dot paper, as illustrated on the right.



"Find the size of an angle" at this grade level means "find the number of degrees in its measure". If two angles have the same size, we say they are equal (meaning equal in measure), or congruent (meaning a tracing of one will fit exactly on the other).

Summary

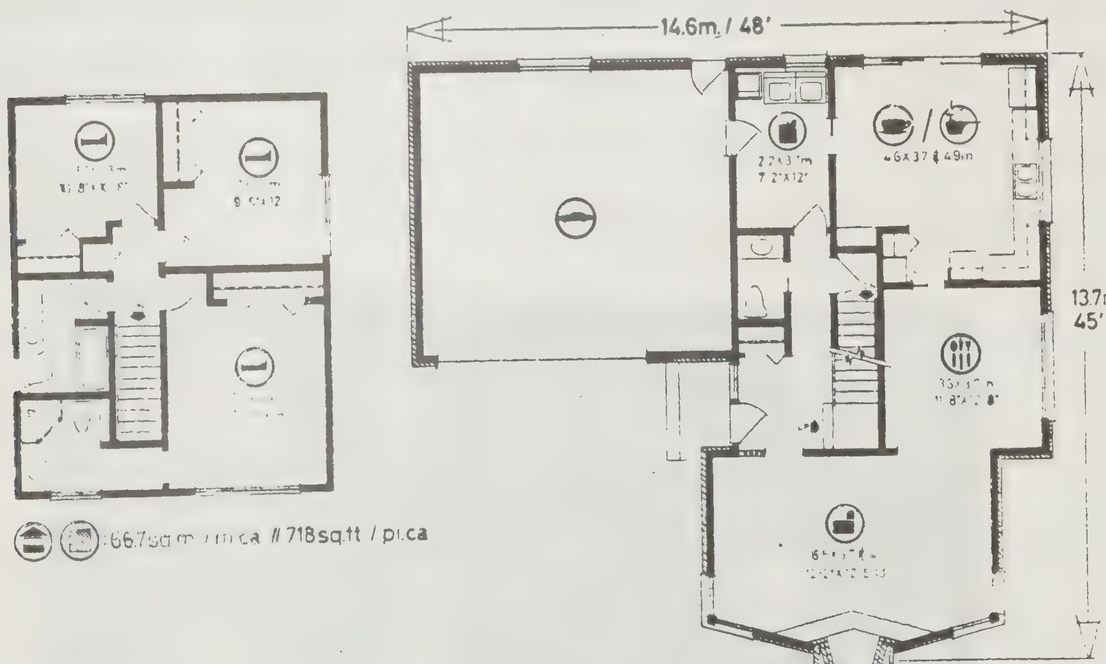
For a given figure (object) and an enlargement or reduction of it (image), the student should know the following:



- The object and image have the same shape but different size.
- The object and image are similar figures
- Corresponding angles are equal.
- The ratios of corresponding sides are equal
- If the enlargement or reduction factor (scale factor) is k, then:
    - the length of a side in the image = k(length of the corresponding side in the object)
    - the area of the image = k^2(area of the object)

b) <u>Scale drawings representing real life situations; scale ratio; converting scale measurements to actual measurements; maps</u>

Scale drawings are used to represent certain features of real life objects or situations. For example, the floor plan of a house (or an apartment) contains significant information if it is drawn accurately to scale -- it gives such information as the width of the windows, the length of wall space on either side of a window, the length of counter on either side of the kitchen sink, the size of the closets (length, depth), the number of stairs, the number of doors and the direction they open.



The scale ratio is important. In the above drawing, it is implied. Find the scale ratio. (Is it the same when found from metric units, and from imperial units? Which is easier to find?) State the scale factor. Use it to find the length of a given piece of wall. Will a certain sofa fit in a certain location?

Draw an enlargement of a given room. Find the dimensions of a choice of furniture to go in the room. Show the furniture on the floor plan.

There are many interesting situations that can be investigated from a floor plan. Ask students to make a floor plan of a room at home, or a part of the house. Consider laying carpet. What are the costs (carpet selected, underpadding selected, labour, cutting the carpet and waste, width of roll, etc.)? What is the cost for drapes?

Suggest that the students make a scale drawing of the front of a wall unit (as in the picture below). Assume the length of the unit is 96" and the height is 72". Make a drawing of the end. Assume the depth of the cabinet is 15". What is the depth of the shelving? What is the scale factor of this photograph?

Scale drawings can be made of a variety of situations in the environment -- furniture, a part of the playing field, buildings, etc. In this case, the students will need to measure the real object, choose a scale, then make an accurate diagram to this scale. Using grid paper will greatly simplify the process. Scale drawings are readily available for a variety of objects in the environment. Obtain some of these. Using the scale, have the students calculate the real dimensions. If the scale is not known, have the students determine the scale by measuring the same dimension on the real object and on the scale drawing.

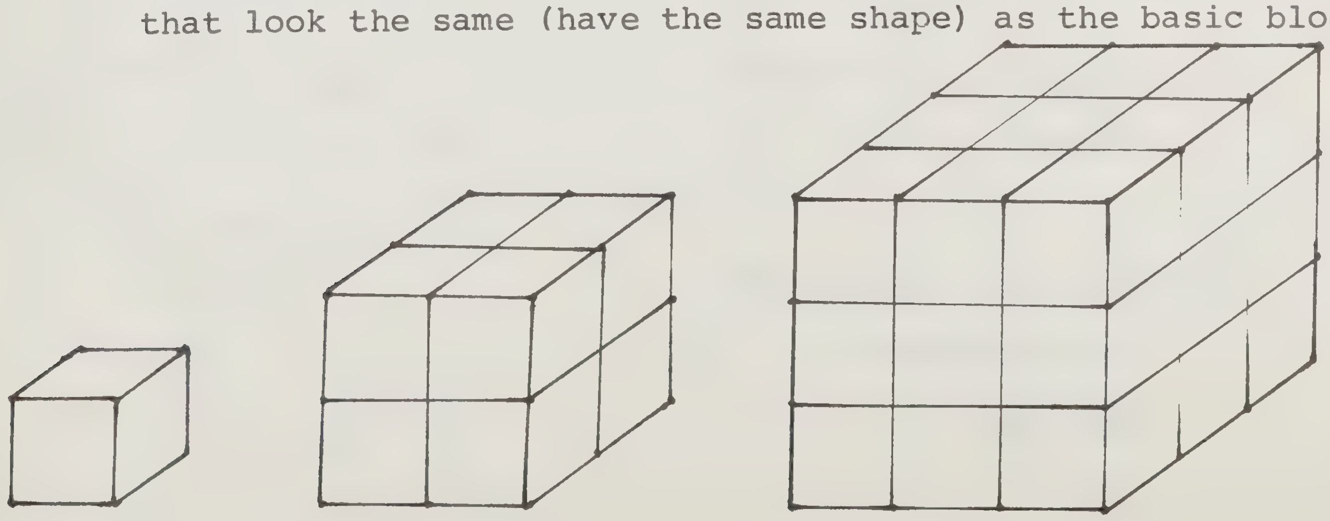
Examine some maps and navigation charts. Note the scale. Find approximate distances.

c) Similar solids

This topic extends the earlier work in topic a) on similar (plane) figures.

Activity 1 (Provide each student (or a group of students) with a set of blocks (cubes).)

i) Stack the blocks, face to face, to make larger solids that look the same (have the same shape) as the basic block.



ii) Compare the lengths of the edges of the blocks. (1:2:3)
Compare the areas of their faces. (1:4:9)
Compare the volumes of the blocks. (1:8:27)

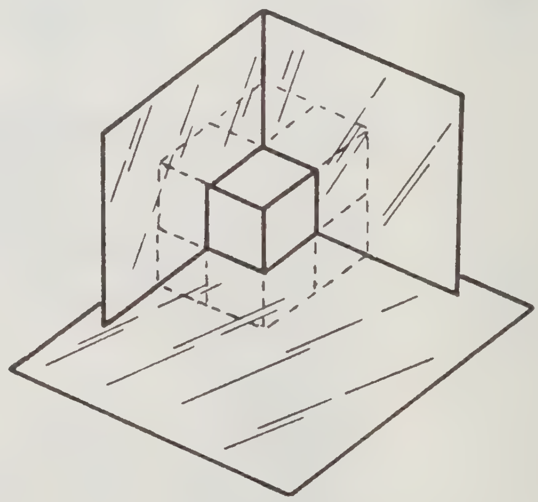
iii) If the next larger block was made, what would be the area of each face, its volume? Generalize on this pattern for k blocks along each edge.

Activity 2 Repeat Activity 1 using rectangular blocks. Are there other solids that will stack to illustrate the same pattern?

Note: If the linear dimensions of a solid are doubled, its volume is eight times greater. The original solid and the new solid are similar.

Activity 3 (This activity should be set up as a demonstration for students to observe and 'putter' with at their own time.)

i) Use 3 mirrors to form a corner, as shown. Tape the mirrors together.



ii) Place a block in the corner. Observe the image.

iii) Make the comparisons as suggested in Activity 1.

iv) Repeat ii) using an 8-block cube.

v) Repeat ii) using a rectangular block.

vi) Investigate the images of other objects.

Similar solids have the same shape. Again, we must use an intuitive feeling for shape at this level. The figures 'look alike'. There are many examples of similar objects in the commercial packaging found in supermarkets. Some of these are in the form of rectangular blocks in different sizes, but there are many other examples such as tooth paste tubes, containers for some dishwashing detergents and antiseptics. Obtain some of these in different sizes; compare one linear measure (say the height) to obtain the scale factor. Use it to compare the volumes of the containers (volumes are in the ratio $1:k^3$ when k is the scale factor). Do these comparisons agree with the volumes printed on the labels?

Ministry of Education
Ontario

GRADE 7 GEOMETRY

SECTION 6: THREE-DIMENSIONAL GEOMETRY

RELATED SECTIONS AND TOPICS

PAST	FY:	Pages 7 and 12; solids -- such as cone, cylinder, sphere, cube, cuboid, prism, pyramid, face, edge, vertex; construct some solids; stacking solids in space; symmetry of 3-D objects
	Ed PJ Div:	Pages 74-76
PRESENT	Gr 7:	N 8; A 1b; G 1; G 2; G 3; G 4; G 5
FUTURE	Gr 8:	N 6; A 1b; G 1ac; G 2ab; G 3d; G 4; G 5
	Gr 9 Gen:	N 6bde; G 1; G 2de; G 3ab; G 4
	Gr 9 Adv:	N 5cdfg; A 2d; G 1; G 2cd; G 3a; G 4bd
	Gr 10 Gen:	G 1; G 2
	Gr 10 Adv:	A 2c;G 1ab; G 4; G 6

a) Identifying real world objects as solids, shells, or skeletons

Our physical world is three-dimensional; the objects we see and touch have dimensions of 'length, width, and height (depth)'. Through a study of geometry we are able to develop a better understanding of this physical world and the miriad of objects in it.

Traditionally, the geometry curriculum has dealt largely with plane figures and their properties. Even though these plane figures can be representations of the flat surfaces of these objects, the curriculum has tended to overlook these connections. It is intended that these connections receive greater emphasis during the implementation of this program. For example, the picture below is a perspective view of a house.

There are many examples of plane figures in this artist's sketch. In the real world the windows, sections of the roof, etc. are rectangular in shape; in this sketch these are not rectangular (check the lines for perpendicularity and parallelism). Initially, students should make diagrams that represent the 'head-on' view, without distortion of the figure. Perspective views are suggested for optional study in topic 7G 6d).

This particular topic involves the classification of real world objects into three categories:

- solids in which the interior is completely filled; for example, a wooden plank, cannon ball, or brick;
- shells in which the interior is not filled; for example, an empty box, ping pong ball, a home;
- skeleton in which the object consists of only a frame; for example, a scaffolding, hydro tower, 2 x 4 framing of a cottage, jungle-bars (in the playground).

Students should be encouraged to make a collection to illustrate these classes of objects, both man made and from nature -- the real objects or pictures of them.

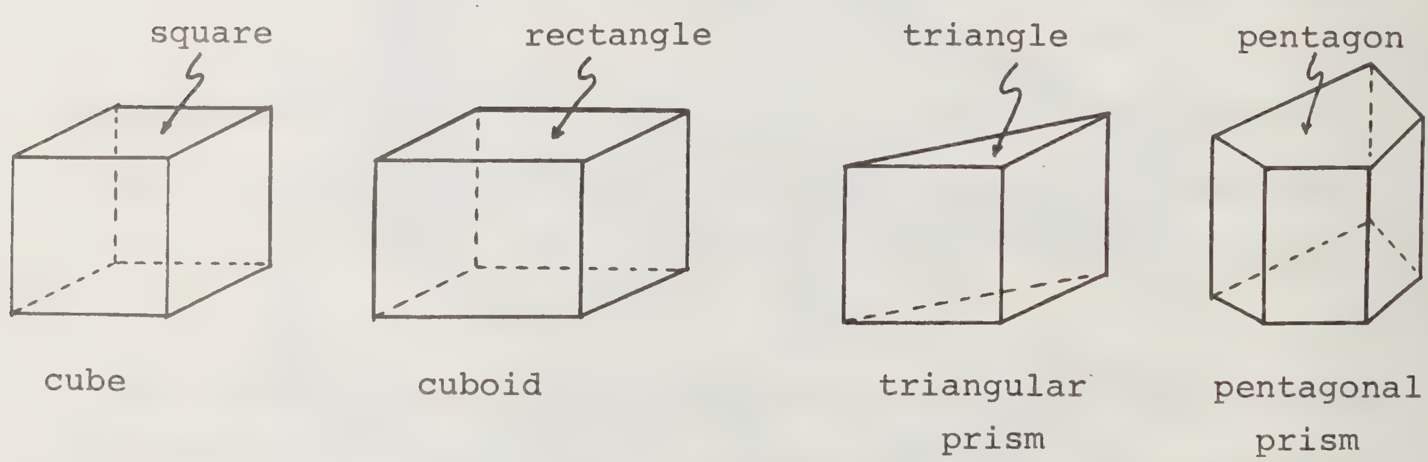
In science, the word 'solid' is used with a different meaning. Understanding should be reached with science teachers on the use of this word in both courses.

Sample physical models of solids, shells (see topic b)), and skeletons (see topic c)) should be on display in the classroom. These should be related to the objects of the environment. Further classification of these models is discussed in other topics in this section.

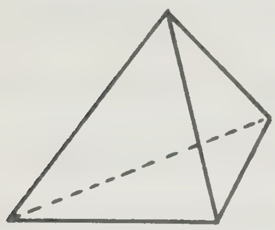
b) Polyhedra in the real world; constructing regular polyhedra from their nets

A polyhedron is a three-dimensional object whose faces are polygons; there are many examples in the real world. Students should be encouraged to identify many of these and to build a classroom collection -- the real objects or pictures of them. Students should be familiar with the terms vertex, edge, and face from their previous studies in the K - 6 program; these terms should be used when discussing the objects. Many of these objects (polyhedra) can be further classified as prisms or pyramids.

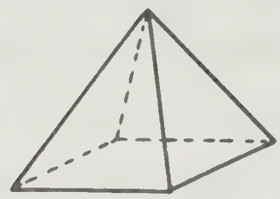
The following are examples of prisms: rectangular box, die, door (plain), wooden plank. In a prism, one pair of opposite faces are congruent and parallel. Prisms are named more specifically by these faces as a cube, cuboid, triangular prism, pentagonal prism, etc.



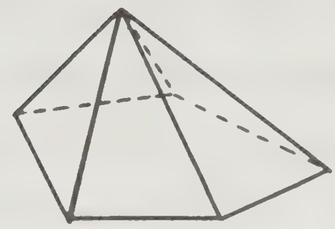
Pyramids are named by their bases.



triangular pyramid



square pyramid

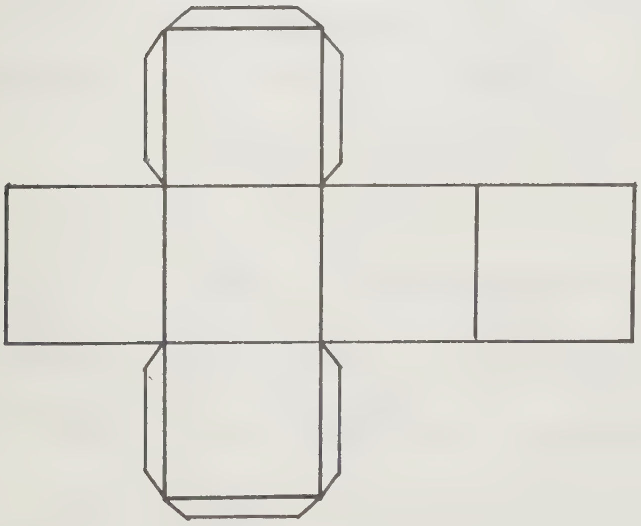


pentagonal pyramid

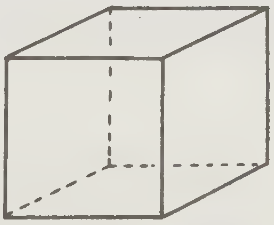
. . . etc.

The roofs (or parts of roofs) of some buildings are pyramids. The tetra pac used for packaging cream or soft drinks is an example of a triangular pyramid (more precisely, a tetrahedron).

The best way for students to become familiar with polyhedra is to build some and handle them. They can be built in a variety of ways. In this topic we are concerned with building shells of polyhedra from their nets. The simplest polyhedron to build is the <u>cube</u>, using a net composed of six squares. One example is given below.



net of a cube



cube

The <u>cube</u> is one of the five <u>regular polyhedra</u>.

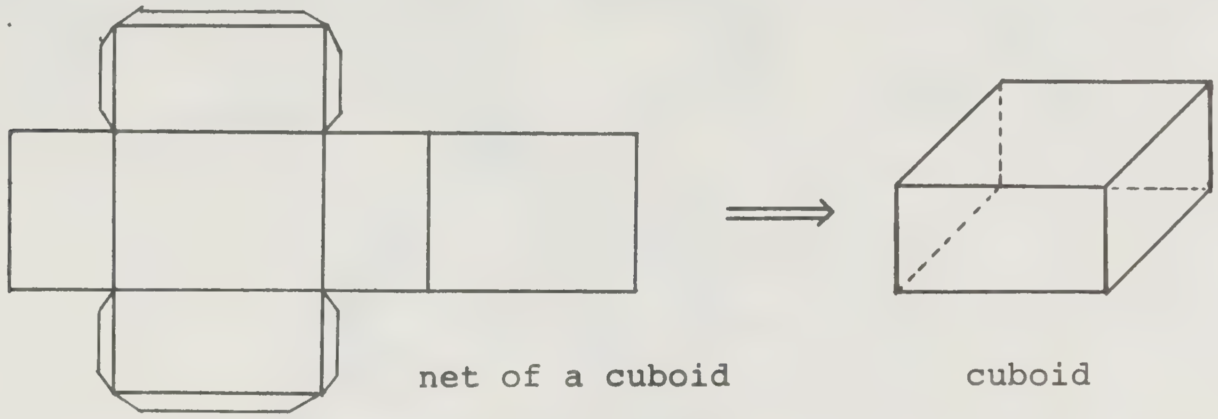
Initially, the students should be given nets that are printed on cover stock or posterboard. The students will cut out the pattern with scissors. Before folding the nets, it is advisable to score each folding edge by firmly tracing a ball point pen against a ruler along each edge. Each student can build two or three of the basic polyhedra quite quickly.

Once the students are familiar with folding of nets, they are ready to investigate ways of building their own nets. They can do this by making a sketch of a net, with a polygon to represent each face of the polyhedron. When convinced that his/her net will work, the student can construct the net and proceed as above. Nets can also be developed by tracing each face of a finished polyhedron, then arranging these 'faces' in a pattern that will fold to form the polyhedron.

As an extension of this activity, the students could be challenged to make as many nets as possible for a given polyhedron. This activity is related to the activities with tiles in 7G 3b)c) in which students were asked to make as many shapes as possible with a given number of congruent tiles.

Many commercial packing boxes are collapsible. Students could bring some to class and investigate the clever ways in which they are constructed. For enrichment, students might investigate flexagons and other non-rigid structures which fold, and fold, and fold.

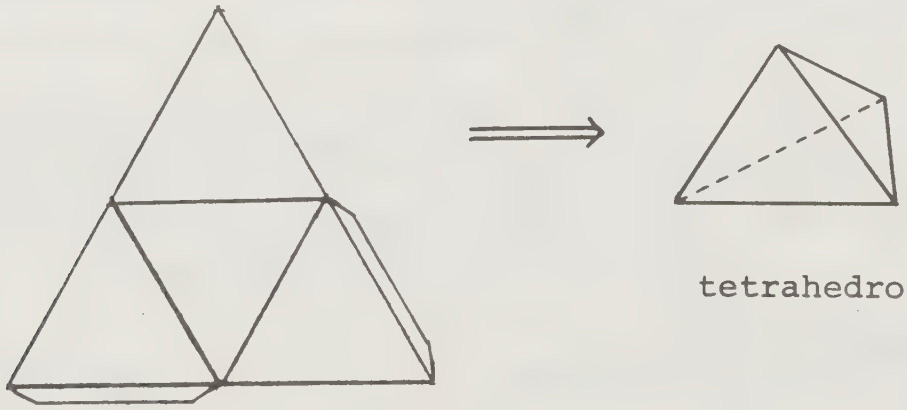
There are numerous shapes for a <u>cuboid</u>; there are many nets for each of these. One example is given below. A cuboid is not a regular polyhedron -- not all of its faces are congruent.



net of a cuboid cuboid

Students might be encouraged to build prisms other than the cube and cuboid.

A regular triangular pyramid is called a tetrahedron. All of its faces are congruent equilateral triangles. A net for a tetrahedron is shown below. Find a different net.



tetrahedron

net of a tetrahedron

This is the shape of the tetra pac referred to above in the notes for this topic.

The other regular polyhedra are:

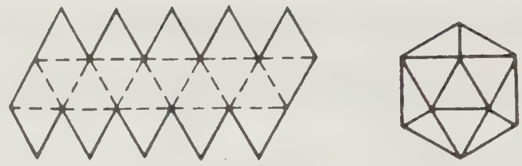
. the <u>octahedron</u> - made from a net of 8 congruent equilateral triangles,

. the <u>icosahedron</u> from 20 congruent equilateral triangles, and

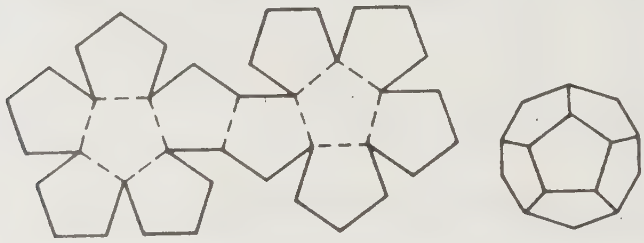
. the <u>dodecahedron</u> from 12 regular pentagons.

Students who are keen on this type of activity could be encouraged to build models of these.

Octahedron



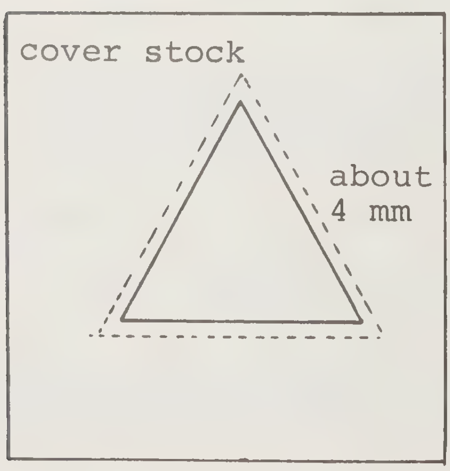
Icosahedron



Dodecahedron

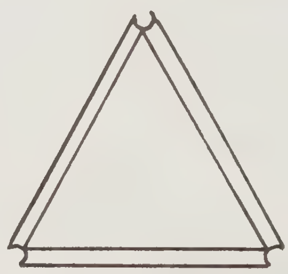
Not all students need do all of the above activities. By having different students construct different polyhedra, models will be available for study by all the class. These should be kept for use when studying topics c), d), and e) below and for work in future grades. Many books, articles, and kits that provide designs for nets of polyhedra, or the nets themselves, are available.

Another interesting way of building models of polyhedra is to cut patterns for each of their faces from cover stock or posterboard, as suggested below for a tetrahedron.



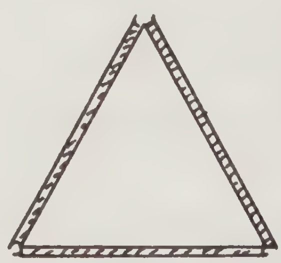
Step 1

Cut along the dashed lines



Step 2

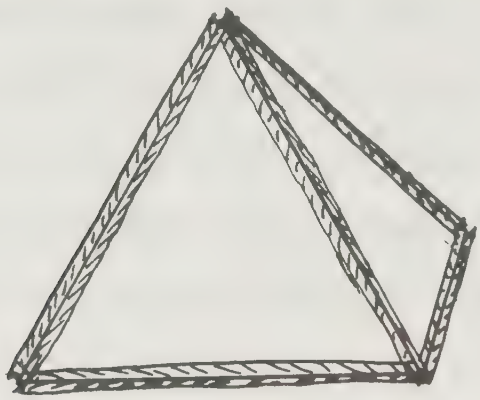
Punch holes at the corners to produce the above figure



Step 3

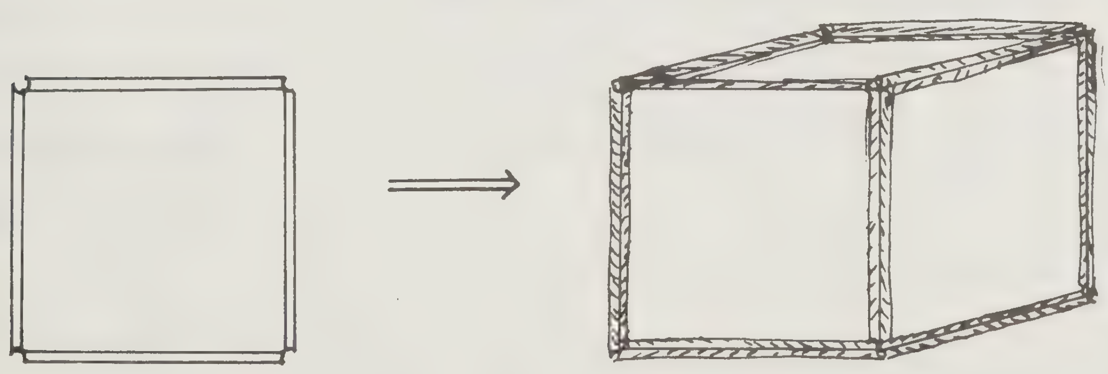
Score the edges of the triangle, fold the tabs up

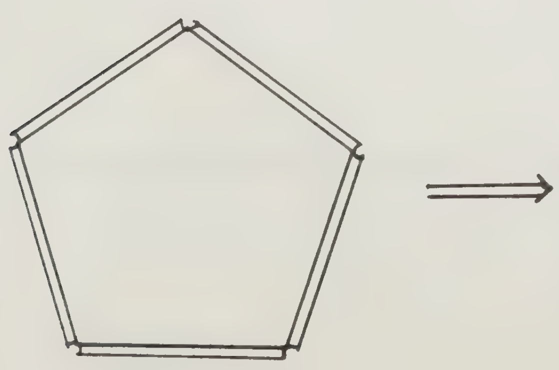
A student (or group of students) will need at least 20 of these congruent equilateral triangles. Join them by rubber bands with the tabs on the outside -- 4 for a tetrahedron, 8 for an octahedron, 20 for an icosahedron.



Ask the students if any other polyhedra can be built from the congruent equilateral faces. There are many, but they are not regular polyhedra. See the note below on regular polyhedra.

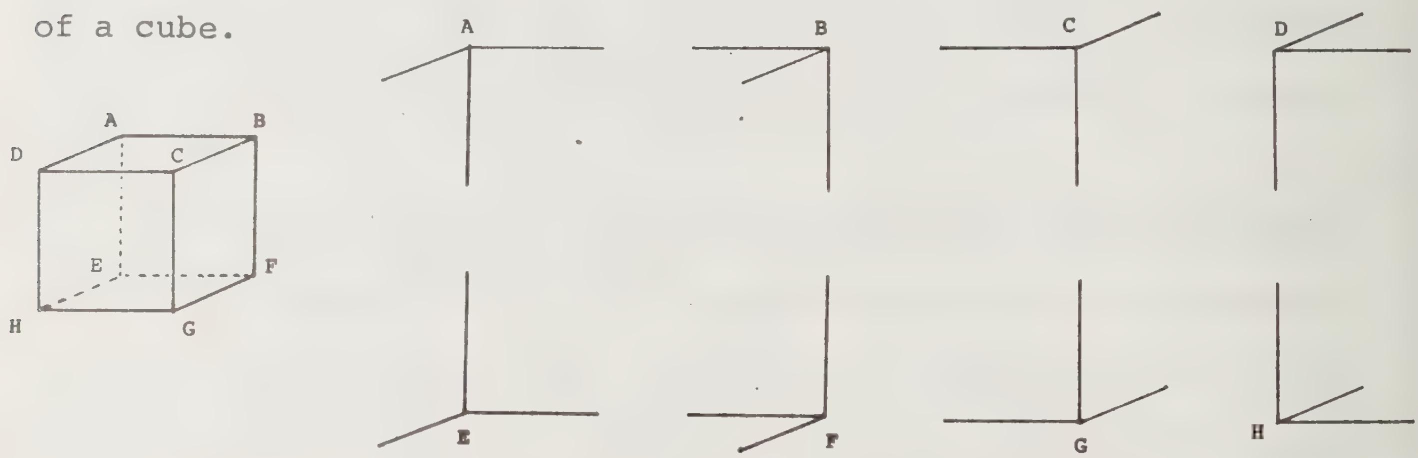
Once the faces have been constructed, the polyhedra can be assembled quickly by the students and then disassembled for use by another group of students. Kits of these faces are available commercially. Have students build different cubes, cuboids, prisms, and pyramids by this method.



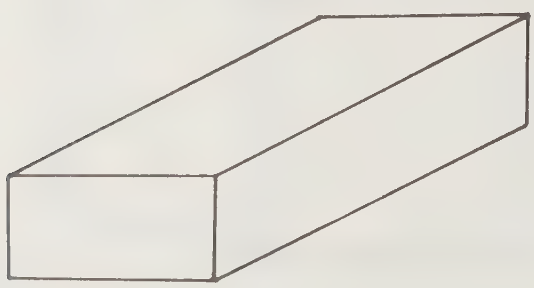


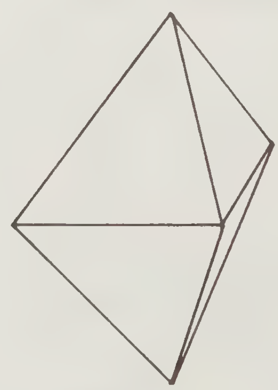
Models of polyhedra may be used to discover Euler's Formula, which relates the number of faces F, vertices V, and edges E of any polyhedron. The formula is $F + V - E = 2$.

A closing comment is in order on the meaning of regular polyhedron. A regular polygon has all sides equal and all angles equal. The faces of a regular polyhedron are congruent regular polygons. A second condition is needed for the polyhedron to be regular -- the configurations at each vertex must be the 'same' (congruent). This condition requires an intuitive sense. It is easier to show they are not the same, than it is to show they are. For example, the configurations are identical at each vertex of a cube.



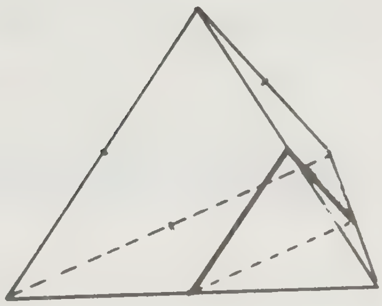
The configurations are not the same at each vertex for a cuboid or the polyhedron, as shown below.





This second condition can be determined more mathematically, as follows.

Locate the mid points of each edge. Select a vertex; join the mid points of the successive edges that emanate from this vertex. This figure must be a polygon.



(This is shown on the right for one vertex of a tetrahedron.) Repeat this process for each vertex. All of the figures so formed must be regular polygons of the same type. If they are, the second condition has been met and the polyhedron is regular.

This second condition for a regular polyhedron is likely too complicated for most students to understand at this age. It has been mentioned only to illustrate why a formal definition of regular polyhedron is not expected. However, students should know for a regular polyhedron that

i) all its faces are congruent regular polygons, and

ii) the configurations at each vertex are the same (more technically congruent).

Students should know that there are only five regular polyhedra. However, it is sufficient, at the core level, that they be familiar with the two simplest cases -- the cube and tetrahedron.

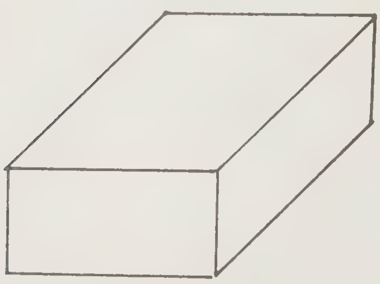
c) Constructing skeletons of pyramids and prisms

This topic is optional in grade 7, but core in grade 8. It is discussed in the notes for 8G 5a).

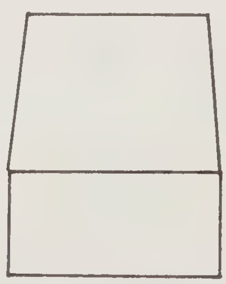
d) Sketching 3-D objects from different perspectives

This topic is optional in grade 7, but core in grade 8. It is discussed in the notes for 8G 5b), and has been discussed briefly in the notes for topic a) of this section.

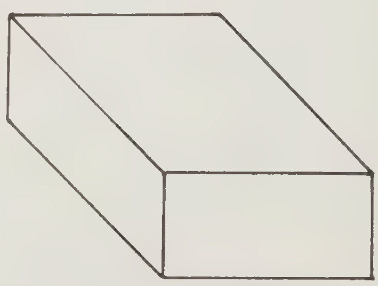
Even though the topic is optional for formal study in grade 7, it is important for students to recognize that most drawings of real world objects are perspective views. This idea should be developed informally when studying the other topics of this section. Students should realize that the objects we see every moment of our waking days change in appearance when viewed from different positions. The drawings we make to represent these 3-D objects distort their true geometric characteristics in order to provide a representation of the 'way it seems to be'. For example, the diagrams that are used to represent a cuboid change with the position from which the cuboid is viewed.



viewed from the right and above



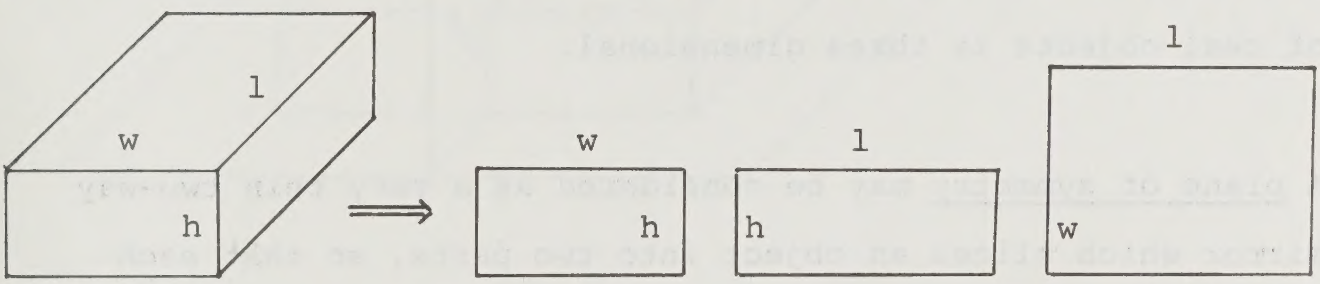
viewed from the front and above



viewed from the left and above

All three angles at each vertex are right angles. In each of the above diagrams, one of the angles is greater than 90^{o}, one equal to 90^{o}, and one less. This is the way it looks.

When a person views a cuboid from directly in front, or from the side, or from above, all that is seen is the one surface at one time, as illustrated.



cuboid   front view   side view   top view

These diagrams are more useful, in the technical sense, since there is no distortion of the lengths of sides or the sizes of angles. There are numerous pictures and diagrams in newspapers, books, and technical journals that illustrate both perspective and 'head on' views of real world objects. Students should be encouraged to look for these and to display examples of them in the classroom.

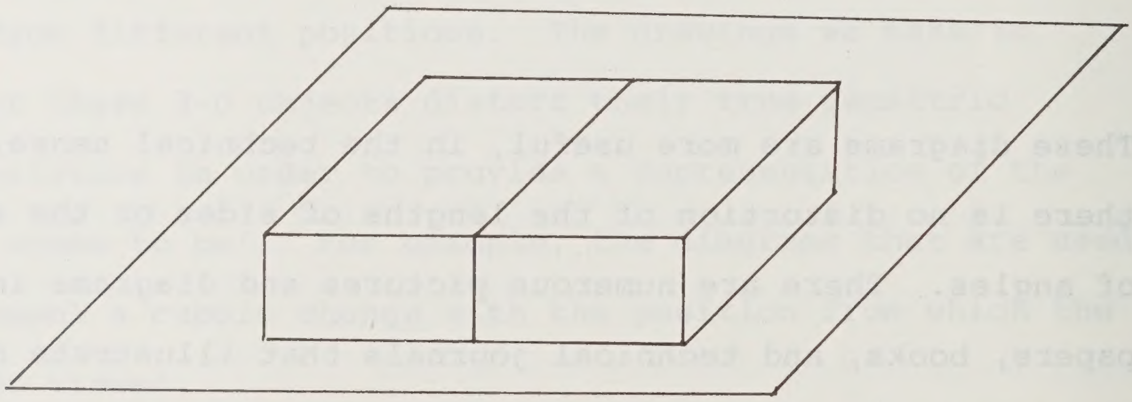
e) Identifying planes of symmetry of 3-D objects

This optional topic is introduced here and extended in 8G 5d) to include axes of symmetry and point of symmetry.

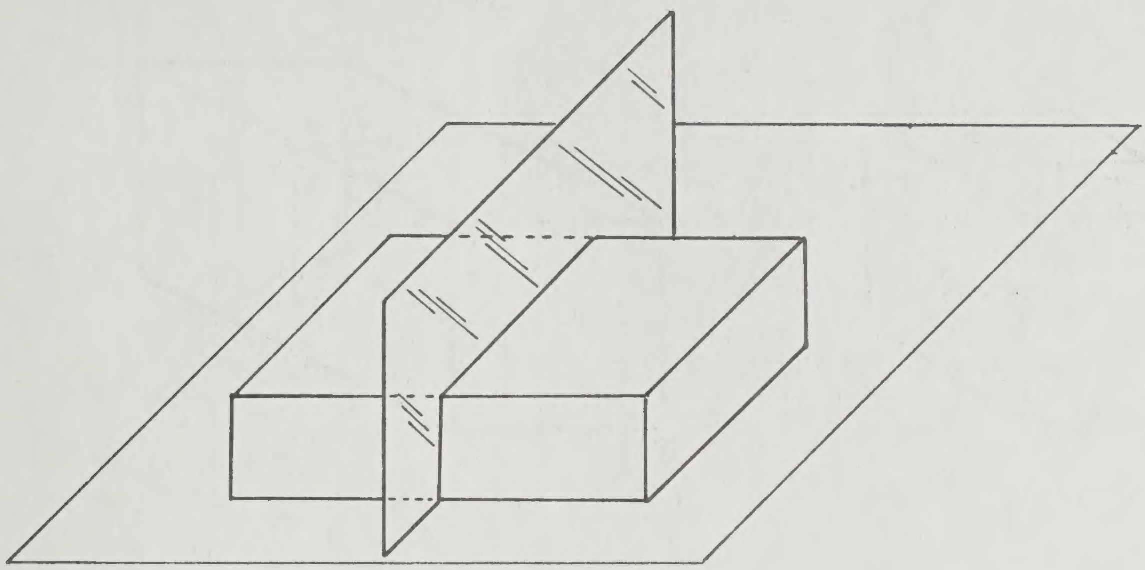
Planes of symmetry

The previous study of line- and rotational symmetry was related to plane figures -- the two dimensional figures of geometry and the flat surfaces of real objects. The symmetry of real objects is three dimensional.

A plane of symmetry may be considered as a very thin two-way mirror which slices an object into two parts, so that each part is the reflection-image of the other. This can be illustrated as follows. Hold two identical blocks of wood together, as if they were one block.



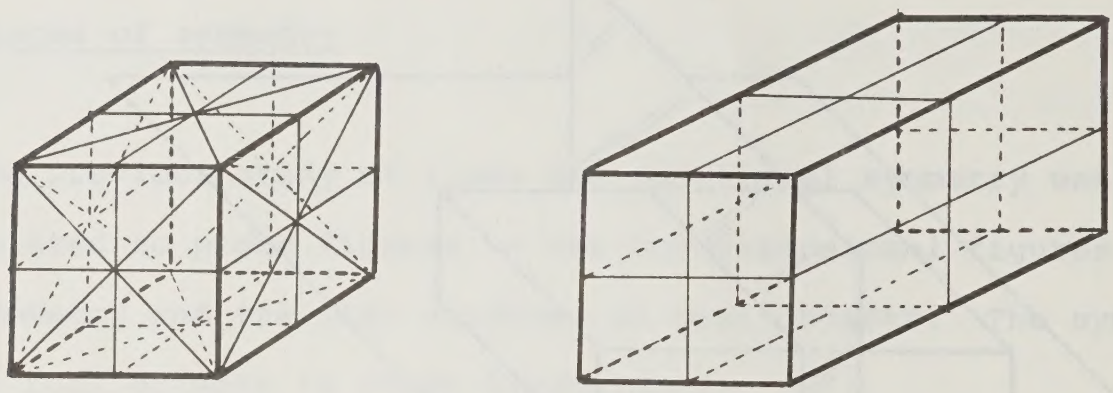
Then separate the blocks by a thin mirror (preferably with a front reflecting surface) as illustrated.



A similar effect can be observed by placing one block against the face of a mirror (front reflecting) and observing that the block-and-image (when thought of as being one figure) has a plane of symmetry.

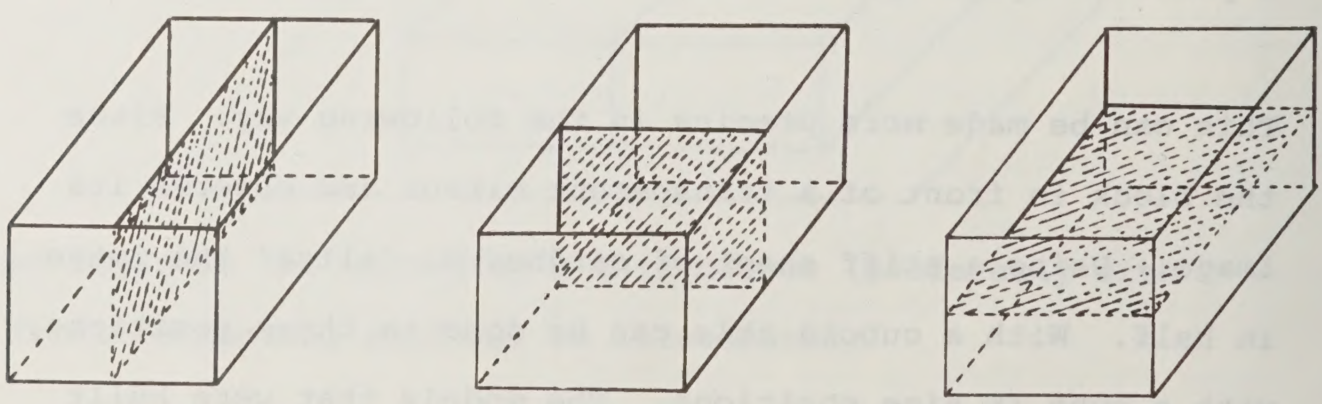
This can be made more precise in the following way. Place the block in front of a transparent mirror and observe its image. Using a stiff sheet of cardboard, 'slice' the image in half. With a cuboid this can be done in three positions, with a cube in nine positions. The models that were built in topic b) can be investigated in this way.

To refine this last activity, draw all the lines of symmetry on each face of the net before folding it. Then fold with these lines on the outside of the polyhedron. These lines, along with the edges, will identify all the planes of symmetry,



Now the image can be sliced by fitting the sheet of cardboard exactly in the position of each plane of symmetry.

Planes of symmetry can be cut from cardboard and fitted into an open box in all possible positions. Students should consider the planes of symmetry of their classroom and compare these to the planes of symmetry of a cuboid and a cube.



It should be noted that real world objects (car, animal, sofa, brick, etc.) have planes of symmetry -- not lines of symmetry. The pictures of these objects have line-symmetry only when viewed 'head on'.